عنوان مقاله

کاربرد مدل‌های یادگیری آماری در بانک‌ها و مؤسسات مالی

- نویسنده

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- **Introduction to Quantitative Finance**
- Introduction to Statistical Learning Models
- Application of Statistical Learning Models in Finance
- Quantitative and Statistical Learning Models for Credit Risk
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Quantitative Finance

- Quantitative Finance tries to find possible models for the following areas:

1) Pricing of Different Contracts
2) Market Risk Management
3) Liquidity Risk Management
4) Credit Risk Management
5) Algorithmic Trading
6) Compliance Risk
7) Investment Management

Pricing of Different Contracts

- These models are mainly mathematical models which are used for the pricing and risk management of sophisticated contracts such as the following:

1) Credit derivatives
2) Equity derivatives
3) Commodity derivatives
4) Fixed Income products
5) FX derivatives

- These models are used in the banks, asset managements, brokerages and insurance companies.

Market Risk Management

- These models are mainly statistical models which tries to capture the risk and volatility in the capital market.
- Possible models in this area are as follow:

1) Value at Risk (VaR)
2) Expected shortfall

These models are used in the banks, asset managements, brokerage and insurance companies.

Liquidity Risk Management

- These models are mainly statistical models which provide the modelling for the following:

1)The behaviour of the balance of assets such as loan, credit cards and overdrafts in the banking book

2)The behaviour of the balance of the liabilities in the banking book.

These models are used in the banks, pension funds and insurance companies.

Credit Risk Management

- These models are mainly statistical models which tries to capture the credit risk and default risk in the trading book and banking book.
- Possible models in this area are as follow:

1) Reduced model
2) Rating Migration Model
3) Merton Models
4) Correlation models
5) Statistical learning Models

These models are used in the banks, asset managements, brokerages and insurance companies.

Algorithmic Trading

- These models are mainly statistical models which provide the modelling for the following:

1) Buy and Sell signal
2) Order execution and pricing

These models are used in the brokerage and asset management companies.

Compliance Risk

- These models are mainly statistical models which provide the modelling for the following:

1) Transaction monitoring
2) Fraud Identification

These models are used in the banks.

Investment Management

- These models are mainly mathematical and statistical models which provide the modelling for the following:

1) Optimal asset allocation
2) Prediction of the price for different assets

These models are used in the asset managements and pension funds.

Model Risk

- Model Risk function tries to validate the models we mentioned earlier.
  - The model risk function propose alternative models for the following models:

1) Credit Risk
2) Liquidity Risk
3) Market Risk
4) Compliance Risk
5) Algorithmic Trading
6) Investment Management

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Statistical Modelling in Finance

- Statistical Modelling and finding the relationship between financial factors is one of the major elements of risk management in Finance.

- The techniques to statistical analysis and finding the relationship between financial factors can be categorized in two broad categories:

1) Econometric Models
2) Statistical learning (Machine Learning) Models

Econometric Models

- These are statistical based approaches such as linear regression or Auto-regression Moving Average(ARMA).

- The following assumptions need to be considered while using these models:

1)Stationarity of the financial time series

2)Not having collinearity in the time series

3) White noise residual

- Such non realistic assumptions can degrade the quality of predictions

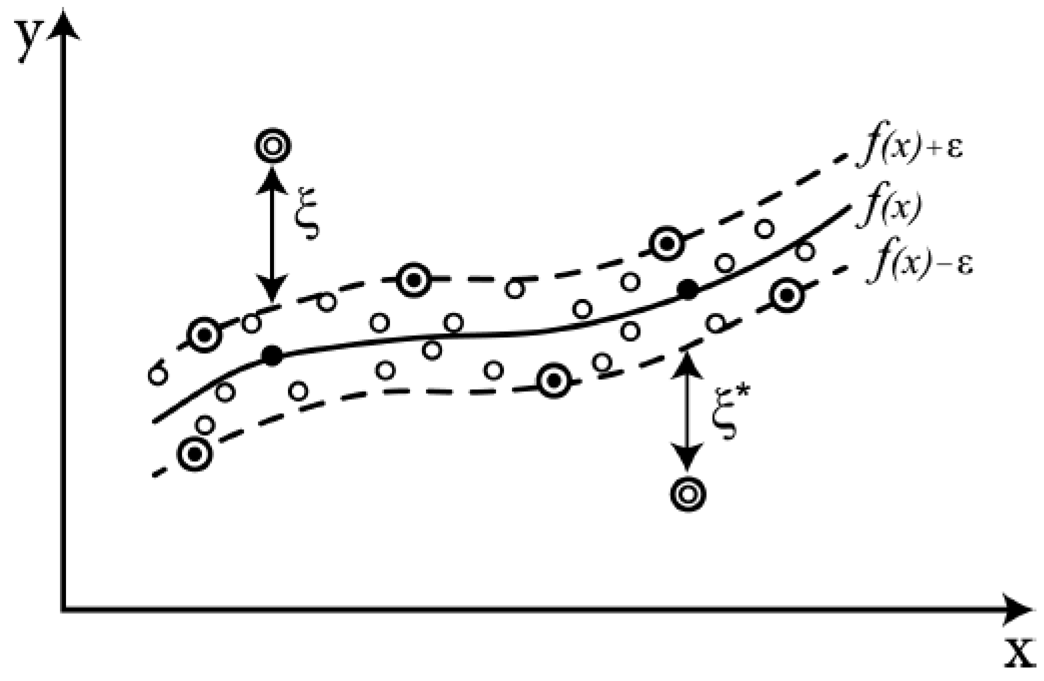
Statistical learning (Machine Learning) Models

- The emergence of artificial intelligence has made it possible to tackle computationally demanding statistical models.

- Some of the machine learning techniques are as follow:

1) Artificial Neural Networks(ANNs)
2) Bayesian Networks
3) Deep Belief Network
4) Support Vector Machine
5) Lasso Regression

Support Vector Regression(1)



Support Vector Regression(2)

- Given a set of data points G= $\{(x_i, y_i)\}_{i=1,\ldots n}$

SVM finds the following equation:

$$f(x) = w\Phi(x) + b$$

- The regression problem can be formulated to minimise the following:

$$R(C) = \frac{C}{n}\sum_{i=1}^{n} L_\varepsilon(f(x_i), y_i) + \frac{1}{2}\|w\|^2$$

$$L_\varepsilon(f(x), y) = \begin{cases} |f(x) - y| - \varepsilon & |f(x) - y| \geq \varepsilon \\ 0 & otherwise \end{cases}$$

Support Vector Regression(3)

$$Min.\ R(w,\xi_i^*) = \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{n}(\xi_i + \xi_i^*)$$

$$\begin{cases} y_i - w\phi(x_i) - b \le \varepsilon + \xi_i \\ w\phi(x_i) + b - y_i \le \varepsilon + \xi_i^* \\ \xi_i, \xi_i^* \ge 0 \end{cases}$$

Support Vector Regression(4)

$$f(x) = \sum_{i=1}^{n} (\alpha_i - \alpha_i^*) K(x_i, x) + b$$

$$\sum_{i=1}^{n} (\alpha_i - \alpha_i^*) = 0$$

$$0 \le \alpha_i \le C \qquad i = 1, \dots, n$$

$$0 \le \alpha_i^* \le C \qquad i = 1, \dots, n$$

$$K(x_i, x_j) = \phi(x_i).\phi(x_j)$$

Polynomial Kernel	$K(x_i, x_j) = (x_i.x_j + 1)^d$
Hyperbolic Tangent Kernel	$K(x_i, x_j) = tanh(c_1(x_i.x_j) + c_2)$
Radial Basis Kernel	p: $K(x_i, x_j) = exp(\|x_j - x_i\|/2p^2)$

Model Inputs for Support Regression

- The input to the models for support vector regression are as follow:

1)Input time series $\{x_i, y_j\}_{i=1,..,n}$

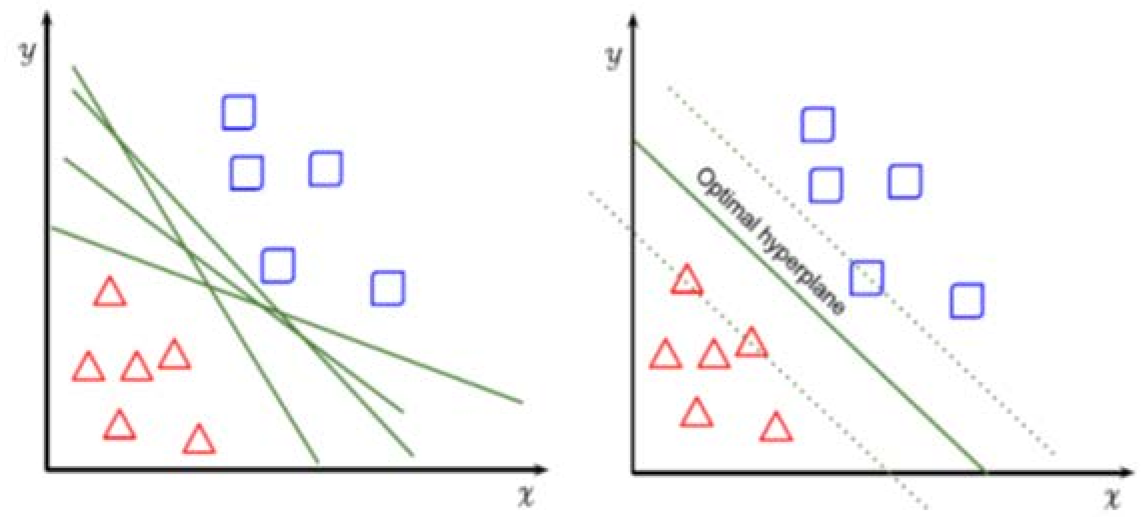
2) Kernel Function

3) ε

4) C

5) The grid search will be performed to find the best parameters given above.

Support Vector Machine-Classification(1)



Support Vector Machine-Classification(2)

Consider the problem of separating the set of training vector belonging to two separate classes, $G = \{(x_i, y_i), i = 1,2, \dots, N\}$ with a hyperplane $w^T \Phi(x) + b = 0$ ($x_i \epsilon R^n$ is the ith input vector, $y_i \epsilon \{-1,1\}$

The original SVM classifier satisfies the following conditions:

$$w^T \Phi(x_i) + b \geq 1 \text{ if } (y_i = 1) \quad (1)$$

$$w^T \Phi(x_i) + b \leq -1 \text{ if } (y_i = -1) \quad (2)$$

or equivalently,

$$y_i[w^T \Phi(x_i) + b] \geq 1 \quad i = 1,2, \dots, N$$

Support Vector Machine-Classification(3)

- We can find the hyperplane that optimally separates the data by solving the optimization problem:
- $min\ \Phi(\mathrm{w}) = \frac{1}{2}||w||^{2}$
- The solution for the above optimization problem is obtained by the saddle point of the Lagrange function
- $L_{P1} = \frac{1}{2}||w||^{2} - \sum_{i=1}^{N} a_i[(y_i(w^T\Phi(x_i) + b) - 1]$

Support Vector Machine-Classification(4)

$$y_i[w^T \Phi(x_i) + b] \geq 1 - \varepsilon_i \quad \varepsilon_i \geq 0 \quad i = 1,2,\dots,N$$

$$min\ \Phi(\mathrm{w}, \varepsilon) = \frac{1}{2}\|w\|^2 + C \sum_{i=1}^{N} \varepsilon_i$$

$$L_{P2} = \frac{1}{2}\|w\|^2 + C \sum_{i=1}^{N} \varepsilon_i - \sum_{i=1}^{N} a_i[y_i(w^T \Phi(x_i) + b) - 1 + \varepsilon_i] - \sum_{i=1}^{N} \mu_i \varepsilon_i$$

Support Vector Machine-Classification(5)

The non-linear classification function is

$$f(x) = Sighn(\sum_{i=1}^{N} a_i y_i K(x_i, x) + \frac{1}{N_s} \sum_{0 \le a_i \le C} (y_i - \sum_{j=1}^{N} a_j y_j (\Phi(x_j)^T \Phi(x_i)))$$

Model Inputs for Support Vector Machine-Classification

- The input to the models for support vector machine in classification mode are as follow:

1)Input time series $\{x_i, y_j\}_{i=1,\dots,n}$

Note that the output are in the binary mode(+1,-1)

2) Kernel Function

3) *C*

4) *p*

- The grid search will be performed to find the best parameters given above.

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Areas of Application of Statistical Learning Models

- Some of the areas where machine learning techniques can be applied in the Finance and Risk are as follow:

1) Credit Risk
2) Liquidity Risk
3) Market Risk
4) Compliance Risk
5) Model risk
6) Algorithmic Trading
7) Investment Management

Application in Credit, Liquidity, Market and Compliance risk

- For Credit Risk, Liquidity Risk, Market risk and Compliance risk the following can be performed by statistical learning model:

- Data Cleaning

- Data classification

- Finding the relationship between risk and economic factors

- Construction of the emerging market data

Application in Model Risk

- For model risk the following can be performed by statistical learning models:
  - We can propose alternative model for the following models:
    1) Credit Risk
    2) Liquidity Risk
    3) Market Risk
    4) Compliance Risk
    5) Model risk
    6) Algorithmic Trading
    7) Investment Management
  - Analysis of documents
  - Data cleaning

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- **Quantitative and Statistical Learning Models for Credit Risk**

Credit Risk

There exist credit risk in the following scenarios

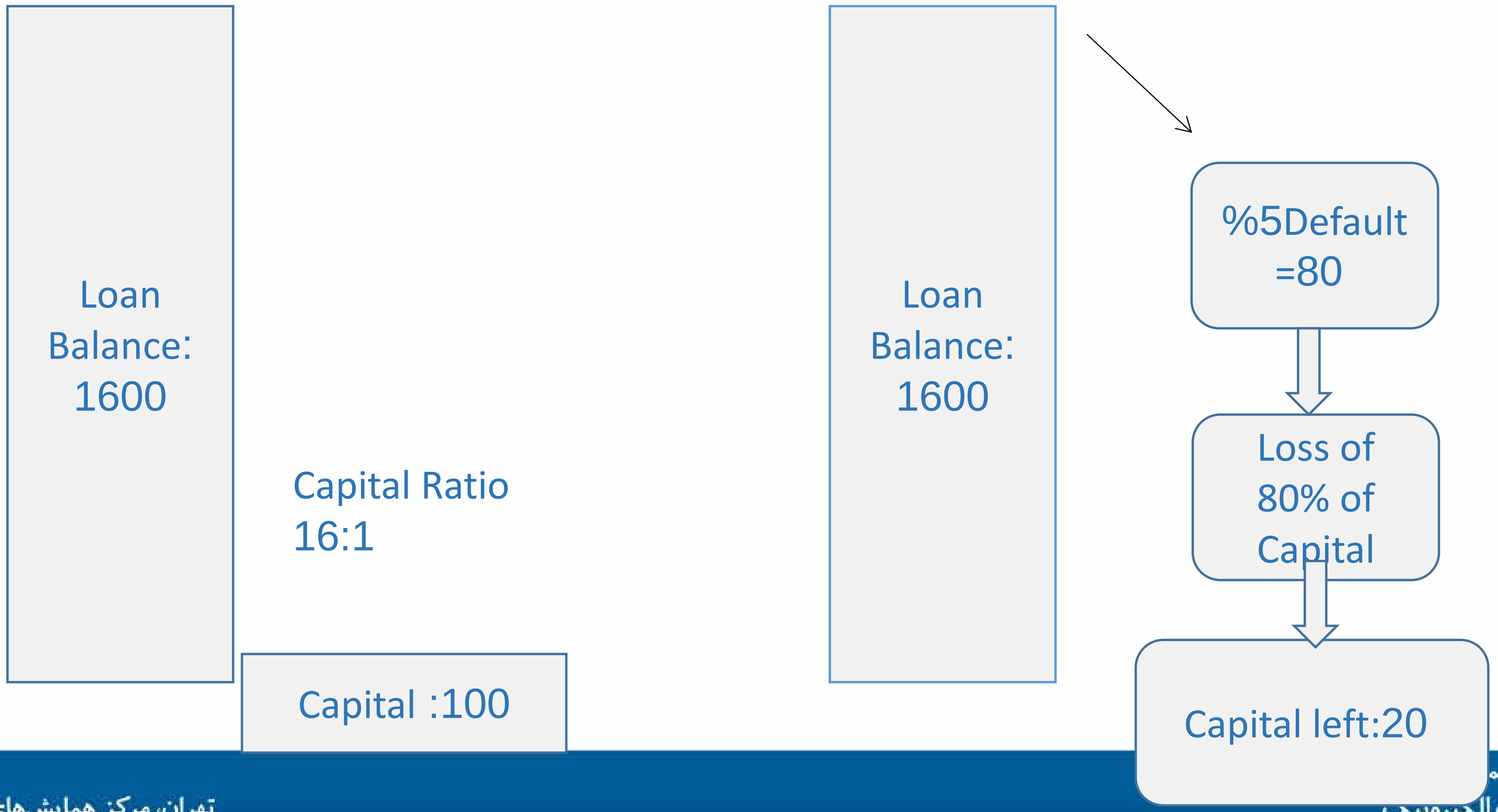
- Bank Loss due to the credit event risk
- Bank Loss due to the change in the credit rating of the customers
- Bank Loss due to the default of the customers

Importance of management and measurement of credit risk

Banks aims to measure and manage credit risk to increase the profit and decrease the loss due to the credit risk

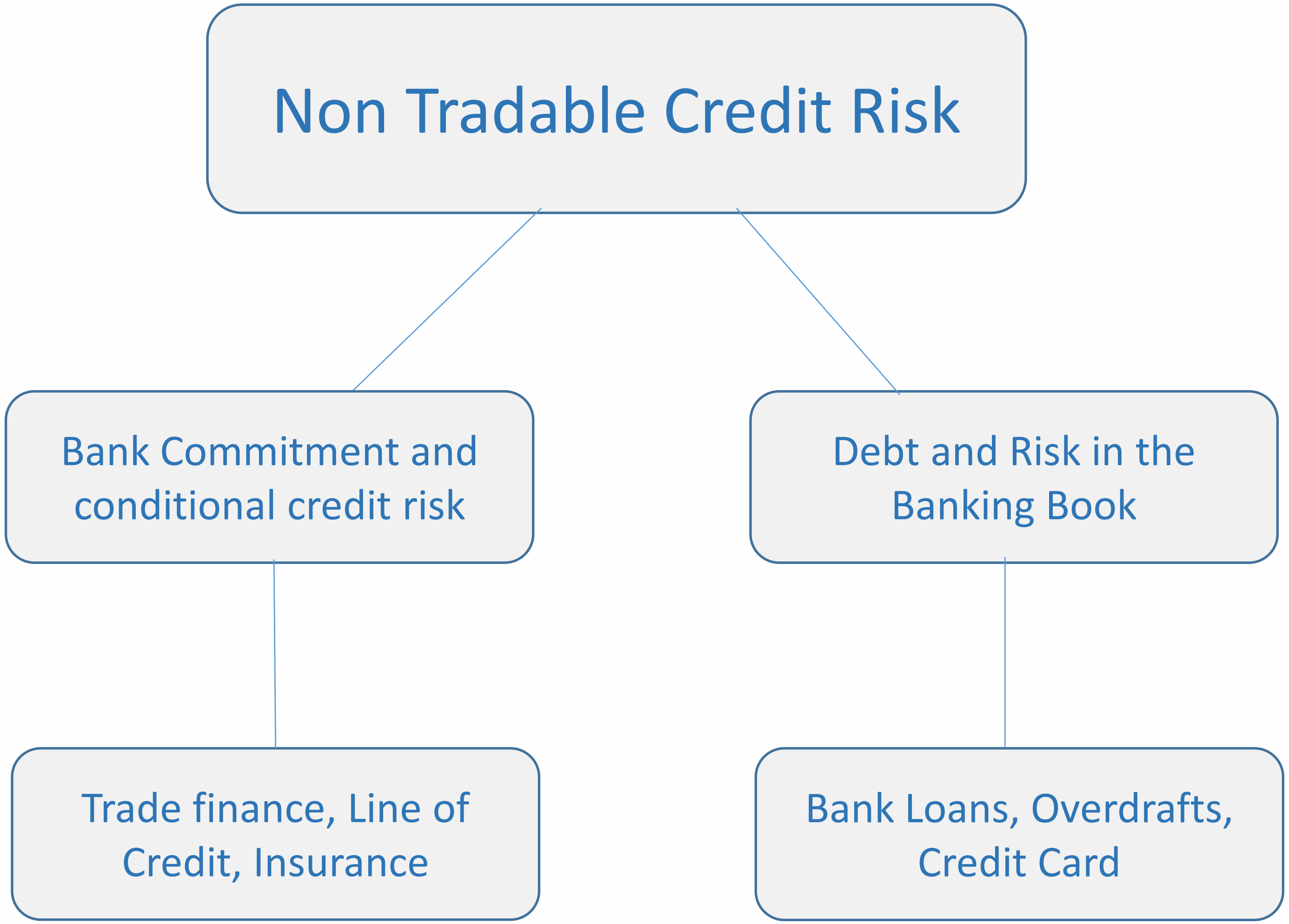
Due to the regulation banks needs to have adequate capital for credit risk.

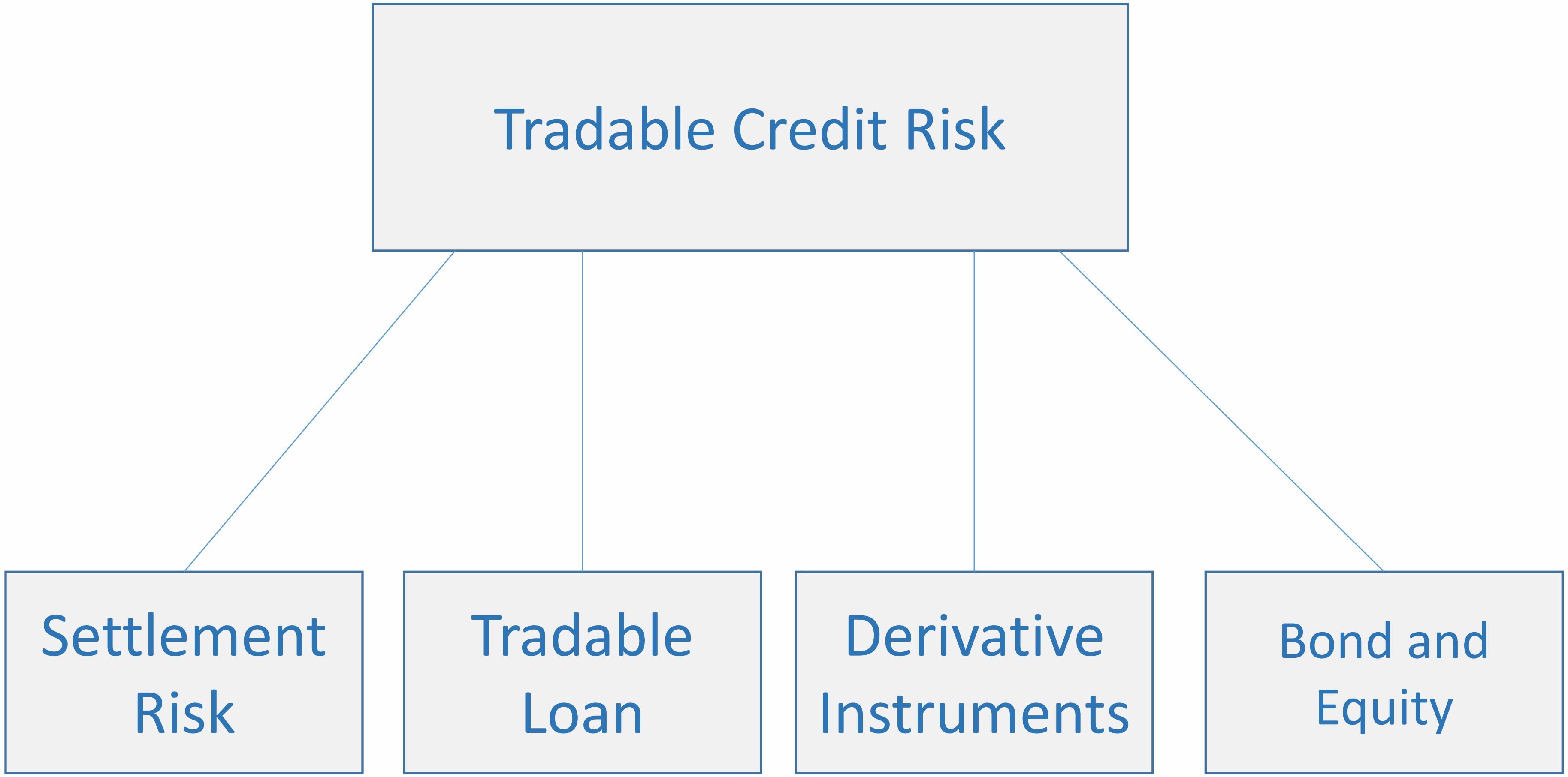
Importance of Credit Risk



Different Types of Credit Risk

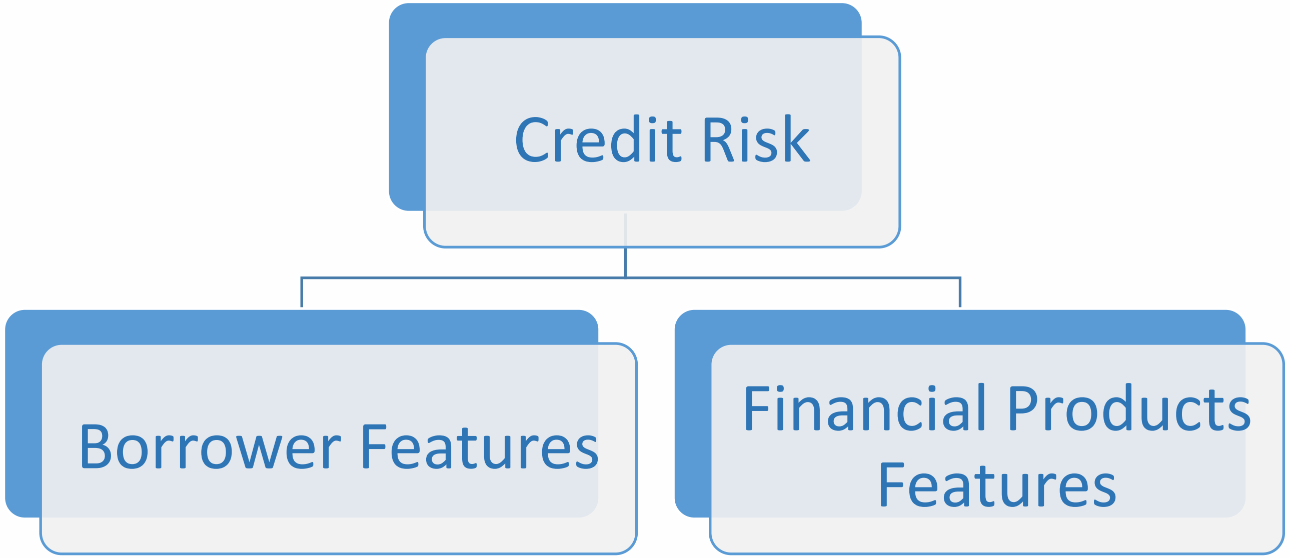
- Non Tradable Credit Risk
- Tradable Credit Risk


Non Tradable Credit Risk
Bank Commitment and conditional credit risk
Debt and Risk in the Banking Book
Trade finance, Line of Credit, Insurance
Bank Loans, Overdrafts, Credit Card


Tradable Credit Risk
Settlement Risk
Tradable Loan
Derivative Instruments
Bond and Equity

Credit Risk Modelling

Variables for Measuring Credit Risk



Borrower Features

Borrower Features

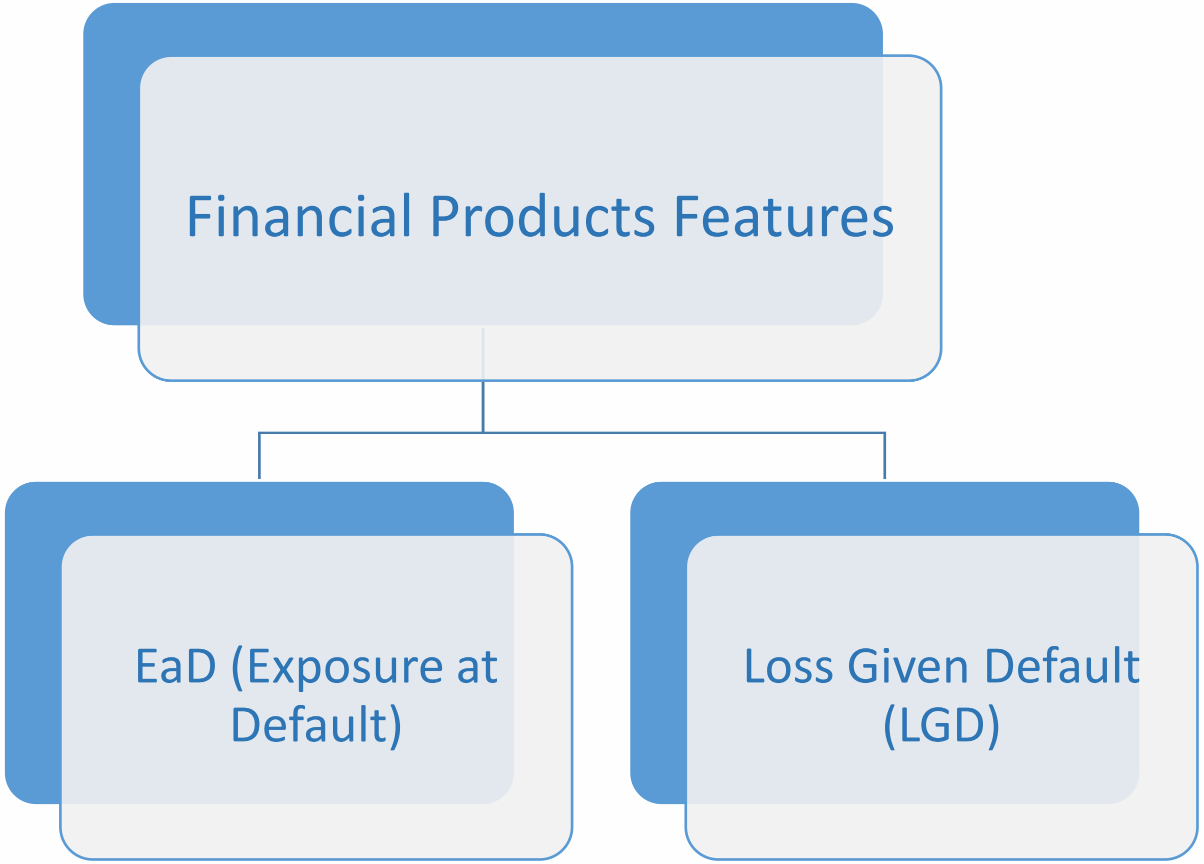
↓

Financial Status of the Borrower

↓

Probability of Default

Financial Products Features



Credit Risk Measurement

Expected Loss= PD × EAD × LGD

PD = Probability of Default

EAD = Exposure at Default

LGD = Loss Given Default

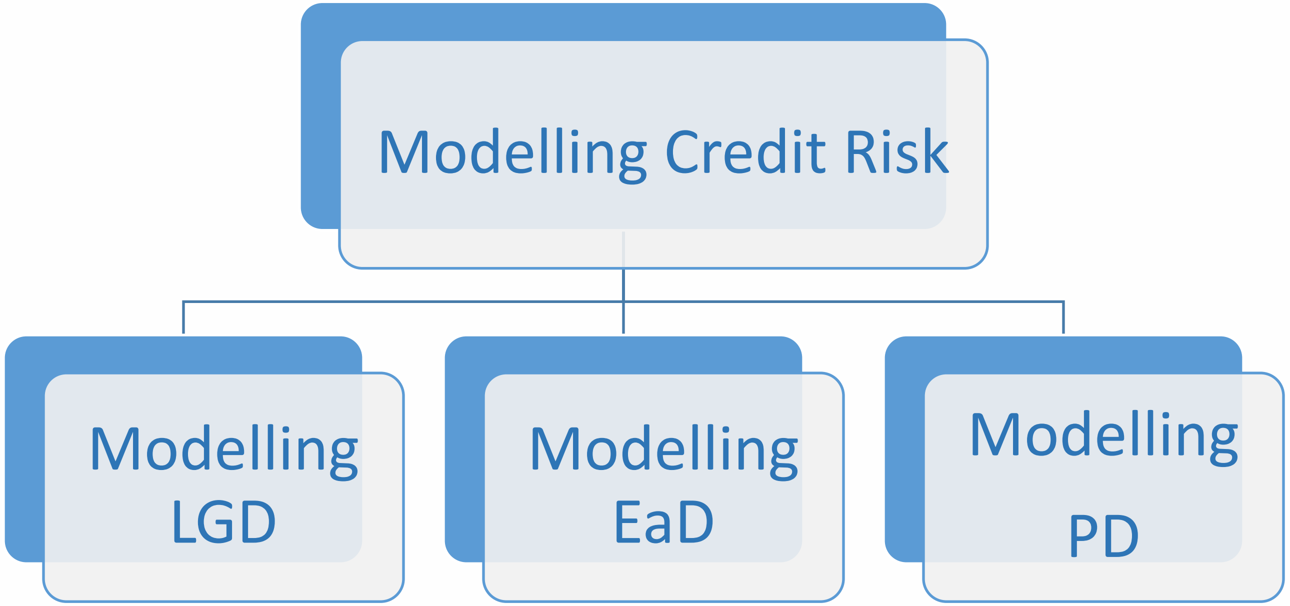
An Example for Expected Loss

	Contarct1	Contract2
Contract	Interest Rate Swap N = 100 m T = 5 year Rate = 5 %	Corporate Loan T = 10 year N = 2m
Probability of Default in 1 year	0.12 % (A rating)	2.3 % (BB rating)
Contract Value at the time of Default	2m Estimation	It depends on the Loan
Expected Loss	50 %	70 %

Challenges in Credit Risk Modeling

- In the past the banks were managing credit risk without measuring it.
- Modelling credit risk is more difficult than market risk.
- Credit risk has skew and fat tail in their statistical distribution and normal distribution model can not model it.
- Correlation between defaults in credit risk is considerable which can not be seen in the case of market risk.
- There are not a lot of data available.
- Change in credit risk happens in time and it is not easily measurable.

Credit Risk Models



Modelling Probability of Default

Retail and small and medium corporates	Regression and Classification Model
Companies that price for their bonds exist	Reduced Models
Finding probability of default from equity price	Structural Merton Model
For big corporates considering rating migration and default	Rating Migration Model
These models show the relationship between the defaults in the basket.	Correlation Model

Regression and Classification Models

$$f(PD) = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4$$

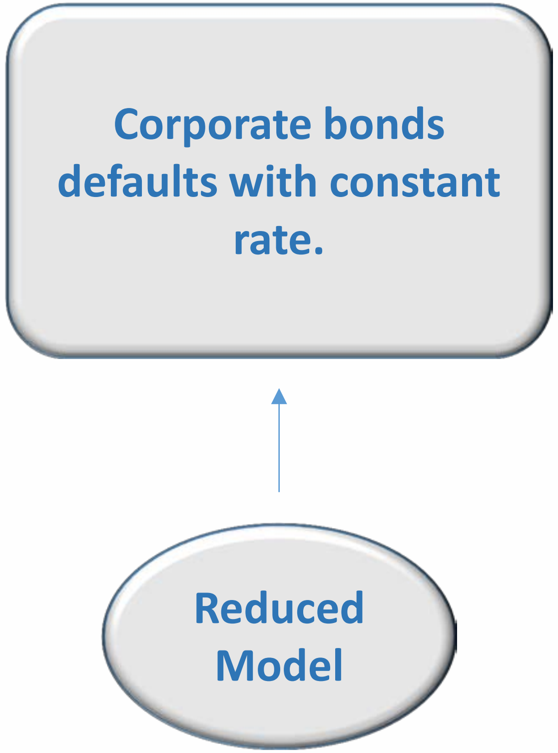
- X1 = Current Assets/ Current Liabilities
- X2= Net Income/Stock Holders Equity
- X3=Equity Value/Debt Value
- X4= Long Term Debt/Total Assets

There exist a lot of variable which are not listed here.

Assumption in Regression and Classification Models

- Independent and dependent variables should be stationary.
- If they are not stationary with some conversion we have to make them stationary.
- There is not collinearity between dependent variables.
- We can use PCA as replacement.
- Error term is a white noise.
- If we use Support Vector Machine for Classification or Regression the condition above does not need to be satisfied.

Reduced Model

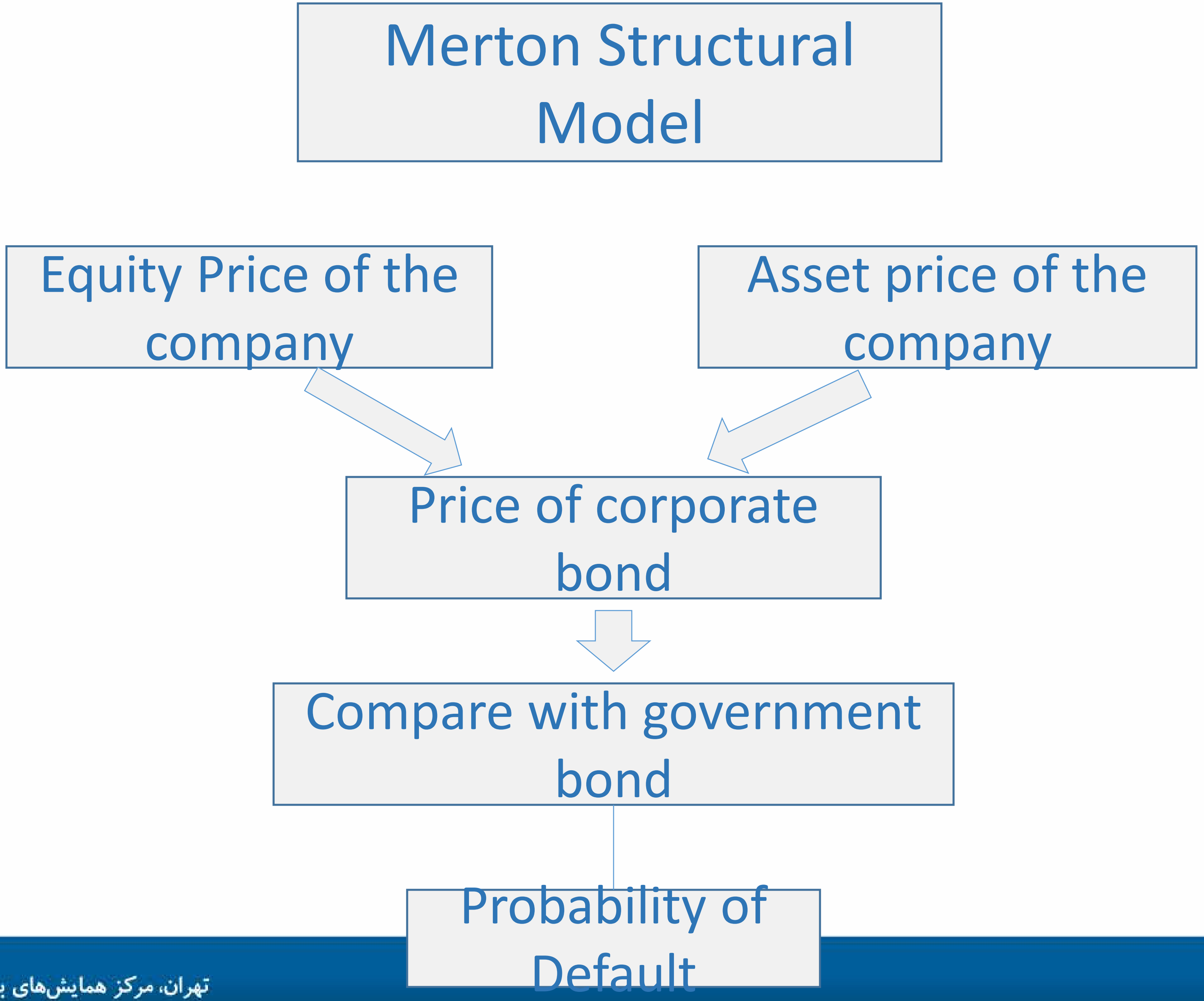


The bond defaults with the rate λ

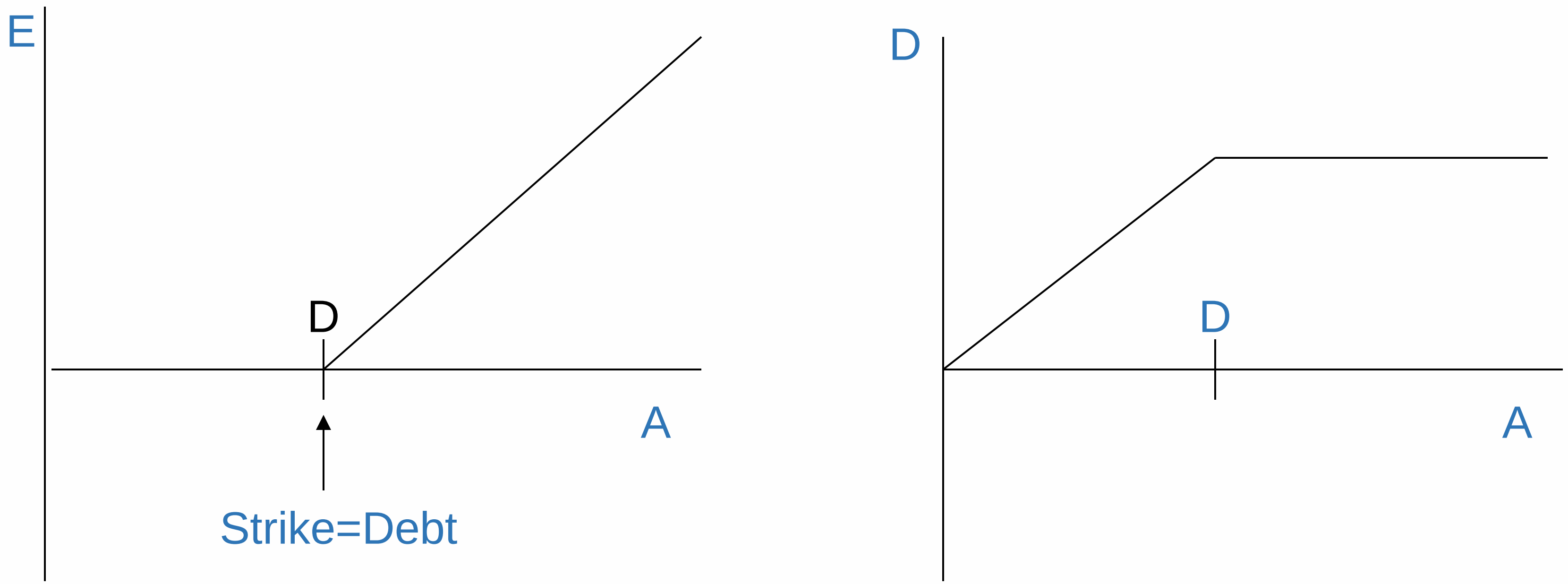
Reduced Model

$$P\_Def(t \leq \tau \leq T) = 1 - \exp\left(\int_t^T -\lambda(s)ds\right)$$

$$P\_Def(t \leq \tau \leq t + \Delta t | \tau \geq t) = \lambda \Delta t$$


Merton Structural Model
Equity Price of the company
Asset price of the company
Price of corporate bond
Compare with government bond
Probability of Default

Merton Structural Model



Equity Price is the same as call option on the undelying assets.

Corporate bond price is the same as selling call option and buying underlying assets.

Merton Structural Model

$$Equity\,(t) = \left(Asset\,(t) - Debt(t)\right)^{+}$$

$$Bond(t) = Asset\,(t) - Equity\,(t) = Asset\,(t) - \left(Asset\,(t) - Debt(t)\right)^{+}$$

$$E(0) = A(0)N(d_1) - e^{-rT}\,D\,N(d_2)$$

$$d_1 = \frac{Ln\left(\frac{A(0)}{D}\right) + \left(r - \frac{\sigma_A^2}{2}\right)(T-t)}{\sigma_A\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma_A\sqrt{T-t}$$

$$Bond(0) = A(0) - E(0)$$

Calculating Probability of Default in Merton Model

$$PDF = \frac{PV(Government\ Bond) - PV(Corporate\ Bond)}{PV(Government\ Bond)}$$

Rating Migration Model

S&P Rating
AAA
AA
A
BBB
BB
B
CCC

Moody Rating
Aaa
Aa
Baa
Ba
B
Caa

Rating Transition Matrix

	Year End Rating							
Init Rat	AAA	AA	A	BBB	BB	B	CCC	Def
AAA	93.66	5.83	0.40	0.09	0.03	0.00	0.00	0.00
AA	0.66	91.72	6.94	0.49	0.06	0.09	0.02	0.01
A	0.07	2.25	91.76	5.18	0.49	0.20	0.01	0.04
BBB	0.03	0.26	4.83	89.24	4.44	0.81	0.16	0.24
BB	0.03	0.06	0.44	6.66	83.23	7.46	1.05	1.08
B	0.00	0.10	0.32	0.46	5.72	83.62	3.84	5.94
CCC	0.15	0.00	0.29	0.88	1.91	10.28	61.23	25.26
Def	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100

Rating Migration Model for a Few Years

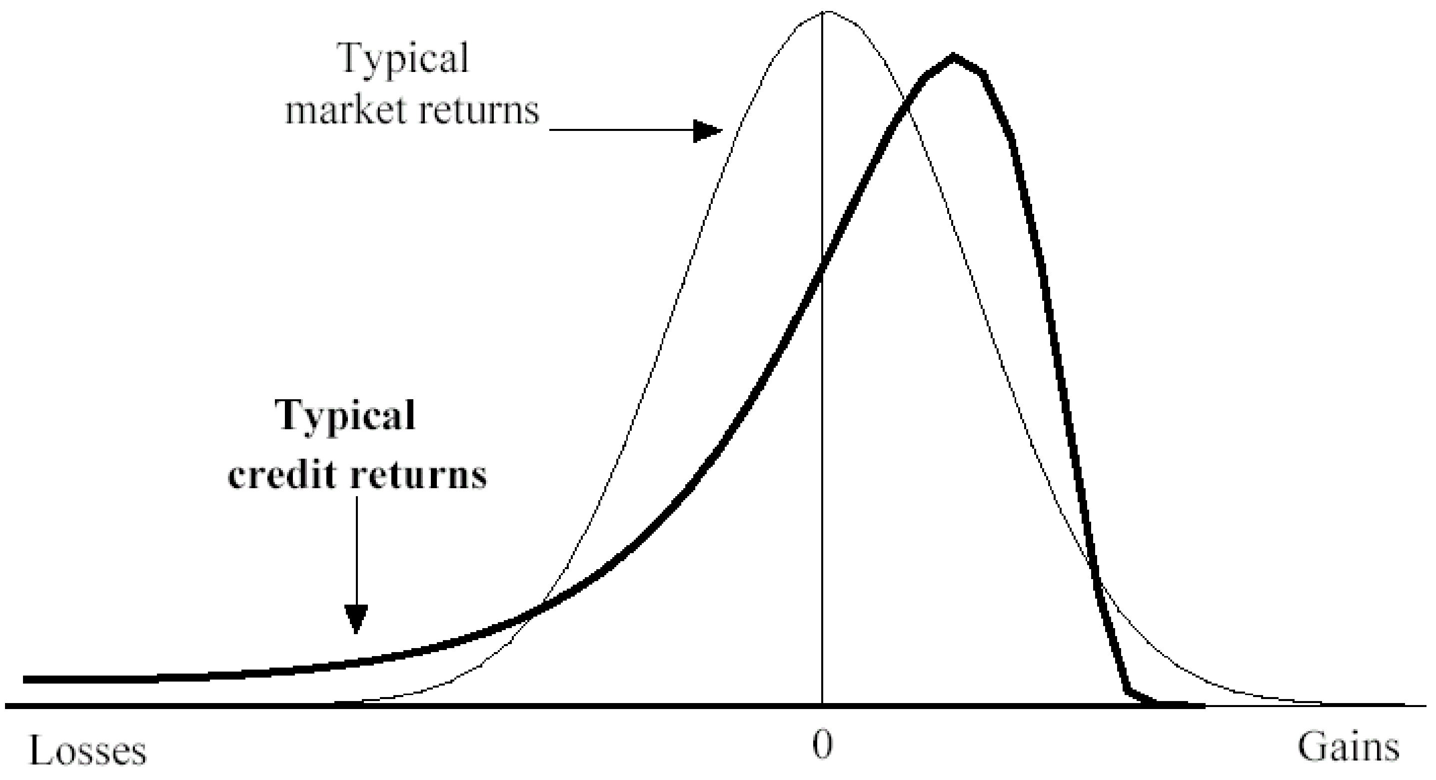
$$A^2 = A \times A$$

$$A^n = A \times A^{n-1}$$

Modelling Correlation between the Defaults

- Correlation of the defaults means that two companies defaults at the same time.
- These correlation often happens in credit crisis.
- This correlation modelling is usually used in pricing credit derivatives such as CDO.
- This correlation creates fat tails in the distribution of the loss.
- Correlation modelling is difficult due to the data issue.
- Correlation usually happens when borrower have exposure to similar risk factors.

Comparison of distribution of credit returns and market returns



Modelling Loss Given Default(LGD)

- Any borrower has certain probability of default.
- However Loss Given Default depends on the contract.
- Loss Given Default at the time of default is not clear even at the time of default.
- Estimation of LGD includes the following:

1) Accurate financial information from the companies
2) Legal aspect of defaults

Modelling Exposure at Default

- The exposure at default can be modelled through the following approach:

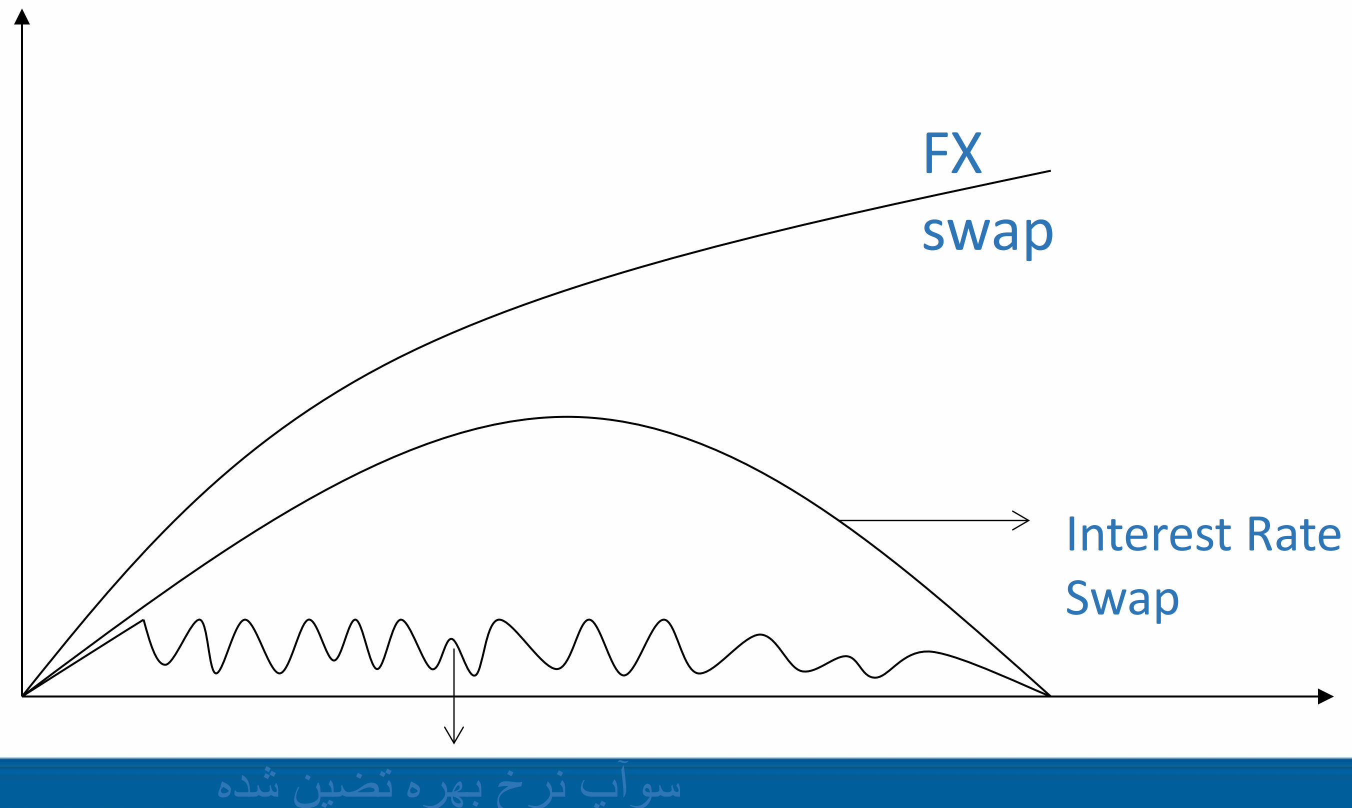
1)Contractual

2)Analytical

3)Monte Carlo Simulation

- The monte carlo simulation is usually used for complicated contracts such as derivatives

Modelling Exposure for Derivative



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