Abstract
One of the key differences between exogenous and endogenous growth models is that a transitory shock to investment share exhibits different long-run effects on per-capita output. Exploring this difference, the present paper evaluates the empirical relevance of the two growth models for the G-7 countries. The underlying shocks are identified by an application of a dynamic factor model. Results show that a transitory shock to investment share permanently increases per-capita output in four countries, offering support to the endogenous growth model. This shock also contributes considerably to accounting for the long-run variability of per-capita output. Overall, the endogenous model is found to be empirically more plausible than previous time series studies suggest.

Key words: Exogenous growth; Endogenous growth; Dynamic factor model
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1. Introduction

In response to various failures of the standard exogenous growth model, Romer (1986), Lucas (1988), Rebelo (1991), and others developed endogenous growth models in which steady growth can be generated endogenously without any exogenous technical progress. Subsequently, testing the relevance of exogenous versus endogenous growth models has been a priority for exploring the determinants of long-run growth. Most empirical studies have focused on cross-country variations, especially with respect to convergence issues. Levine and Renelt (1992) surveyed these cross-section studies and concluded that the robust results reject exogenous growth models. Pack (1994), Solow (1994) and Durlauf et al. (2005) suggest that time series studies can make equally important contributions. However, there are only scant time series studies, and the evidence also tends to favor exogenous growth models. Jones (1993), Kocherlakota and Yi (1996), Lau and Sin (1997), and Lau (2008) found that endogenous growth models are not consistent with data in a number of developed countries.

Intrigued by their empirical rejection of endogenous growth models, this paper revisits the exogenous and endogenous growth debate in a time series context.¹ A structural evaluation of the competing growth models is important for policy design, as well as helping to guide future

¹ A group of studies focus on different policy or predictive implications of endogenous growth models. For instance, Bleaney et al. (2001) tested the role of government expenditure/taxation, while Pyo (1995) focused on human capital as a main source of increasing returns. At the time of writing, an empirical study by Cheung et al. (2012) revisits the association between investment and growth using both cross-sectional and time-series regressions. However, all of these studies are based on standard regressions, and hence are not attempts to structurally evaluate exogenous versus endogenous growth models, bearing little direct relevance to the current paper.
theoretical developments. While there are several variables that characterize long-run growth, investment and output are at the root of both exogenous and endogenous growth models. Hence, we explore how different implications of the long-run behavior of investment and output can be more precisely taken to data in a structural time series framework. Particular motivation is due to Lau (1997; 2008), who shows the time series implications of a Solow-Swan exogenous growth model and a typical AK endogenous growth model. The time series implications provide the analytical basis for the empirical tests explored in this paper.  

To be specific, Lau assumes that log per-capita output and log per-capita investment are $I(1)$ processes, and they are cointegrated with a coefficient vector of $[1, -1]$. The investment share, defined as the ratio of per-capita investment to per-capita output, becomes a stationary process. Lau proceeds to prove that a transitory shock to investment share produces permanent effects on log per-capita output in the endogenous growth model, whereas it only has transitory effects in the exogenous growth model. This distinction was similarly used in King et al. (1988) and Kocherlakota and Yi (1996).  

In this paper, we follow the distinction put forth by King et al., Kocherlakota and Yi, and Lau, and propose how to empirically test the implied differences between the Solow-Swan exogenous and AK endogenous models of growth. The procedure is based on the dynamic factor

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2 In Appendix A, we provide a detailed derivation of the time series implications shown, without derivation, by Lau (1997; 2008). We thank an anonymous referee for making this suggestion.  
3 A more oft-cited distinction between the two growth models was their predictions about whether a permanent shock to investment share can permanently affect the growth rate of per-capita output. In endogenous growth models, a permanent investment share shock can, while in exogenous growth models it cannot. The empirical application of this distinction may encounter some difficulties, however. See Lau (2008) for details.
models of Stock and Watson (1988), Johansen (1991), Kasa (1992), and Escribano and Peña (1994). These models were developed for the decomposition of permanent and transitory components in a cointegrated system. We make a modification to use in the identification of structural shocks. By construction, the long-run response of log per-capita output to a transitory shock in investment share is allowed to be determined by data. It is not restricted to being zero, which is distinct from other competing methods. Because the long-run response is identified without recourse to restrictions, checking whether it is zero can constitute a legitimate empirical test for differentiating between the two growth models. In the paper, we also conduct a structural analysis using the Beveridge and Nelson (1981) decomposition of a type used for vector error correction models by Mellander et al. (1992), Englund et al. (1994), and Fisher et al. (2000). This alternative specification is consistent with the exogenous growth model, given that a transitory shock in investment share is restricted to produce no permanent effects. The results are utilized to check the robustness of those from the dynamic factor model.

The remainder of this paper is organized as follows. Section 2 discusses the long-run time-series properties of Solow-Swan exogenous and AK endogenous growth models. Section 3 presents the dynamic factor model for examining the empirical consistency of the two growth models with actual data. Section 4 provides the test results for the G-7 countries and discusses policy implications. Section 5 conducts a Monte Carlo experiment to perform a diagnostic check
on how well the dynamic factor model performs in recovering the long-run responses of the variables. Section 6 summarizes the major findings of the paper with concluding remarks.

2. Theoretical models

Drawing on Lau (1997; 2008), this section illustrates a stochastic Solow-Swan model with exogenous technological processes and, as its endogenous counterpart, a stochastic AK model of the type suggested by Rebelo (1991). Consider a closed economy which is populated by a constant number of identical agents N. The supply side of the economy is represented by a Cobb-Douglas production function:

\[ Y_t = AK_t^{\lambda} N^{1-\lambda} \eta_t^P, \tag{1} \]

where \( Y_t \) is output at time \( t \), \( K_t \) is capital input at time \( t \), \( 0 < \lambda < 1 \), and \( \eta_t^P \) is an impulse process to the otherwise constant level of total factor productivity \( A \). The demand side of the economy is represented by:

\[ I_t / Y_t = s \eta_t^I, \tag{2} \]

where \( I_t \) is investment at time \( t \), \( s \) (\( 0 < s < 1 \)) is the average investment share in output, and \( \eta_t^I \) is an impulse to investment share.

The two impulse processes are assumed to have the form:

\[ (1 - L)^\tau P (L) \ln \eta_t^P = \varepsilon_t^P \quad \text{and} \quad (1 - L)^\tau I (L) \ln \eta_t^I = \varepsilon_t^I, \tag{3} \]
where L is the lag operator, \( \pi_j \) is either 0 or 1, \( Q_j(L) \) is a polynomial function in L with all roots outside the unit circle, and \( \varepsilon^P_t \) and \( \varepsilon^I_t \) are structural disturbances and are assumed to have a mean of zero and an identity covariance matrix. The impulse process \( \ln \eta_t \) is \( I(0) \) when \( \pi_j = 0 \) and \( I(1) \) when \( \pi_j = 1 \). The level of capital stock evolves over time according to

\[
K_{t+1} = (1 - \delta)K_t + I_t,
\]

where \( \delta (0 < \delta < 1) \) is a constant rate of depreciation.

In the Solow-Swan exogenous growth model, log per-capita output and log per-capita investment are \( I(1) \) inherited from an \( I(1) \) process of productivity. This requires that the productivity and investment share impulses are \( I(1) \) and \( I(0) \), respectively, and hence, \( \pi_P = 1 \) and \( \pi_I = 0 \) in (3). The log-linearized equations of motion near the steady-state growth path result in the following vector moving average (VMA) system:

\[
\begin{bmatrix}
(1-L)\ln y_t \\
(1-L)\ln i_t
\end{bmatrix} = \begin{bmatrix}
\delta LQ^P_t^{-1}(L) & \lambda \delta (1-L) LQ^I_t^{-1}(L) \\
\delta LQ^P_t^{-1}(L) & \delta (1-L) LQ^I_t^{-1}(L)
\end{bmatrix} \begin{bmatrix}
\varepsilon^P_t \\
\varepsilon^I_t
\end{bmatrix},
\]

where \( y_t \) and \( i_t \) are per-capita output and per-capita investment, respectively.\(^4\)

The AK endogenous growth model of Rebelo (1991) can be summarized using (2), (3), (4), and

\[
Y_t = AK_t \eta^P_t.
\]

If log per-capita output and log per-capita investment are \( I(1) \), this model implies that both

\(^4\) Appendix A provides a detailed derivation of (5).
productivity and investment share impulses are \( I(0); \) thus, \( \pi_p = 0 \) and \( \pi_I = 0 \) in (3). The log-linearization near the steady-state growth path gives the following VMA formation:

\[
\begin{bmatrix}
(1-L) \ln y_t \\
(1-L) \ln i_t
\end{bmatrix} = \begin{bmatrix}
\ln(1-\delta+sA) \\
\ln(1-\delta+sA)
\end{bmatrix} + \\
\begin{bmatrix}
(1-L) Q_p^{-1}(L) & L \left( \frac{sA}{1-\delta+sA} \right) Q_i^{-1}(L) \\
(1-L) Q_p^{-1}(L) & (1-\frac{1-\delta}{1-\delta+sA}) Q_i^{-1}(L)
\end{bmatrix} \begin{bmatrix}
\epsilon_t^p \\
\epsilon_t^I
\end{bmatrix}
\]

This shows that as long as \( sA > \delta \), the economy grows even without exogenous technological progress.

Both Solow-Swan and AK models of growth in (5) and (7) may be rewritten compactly as:

\[
\begin{bmatrix}
\Delta \ln y \\
\Delta \ln i_t
\end{bmatrix} = \text{constant} + \Gamma(L) \varepsilon_t = \text{constant} + \begin{bmatrix}
\Gamma_{11}(L) & \Gamma_{12}(L) \\
\Gamma_{21}(L) & \Gamma_{22}(L)
\end{bmatrix} \begin{bmatrix}
\varepsilon_t^p \\
\varepsilon_t^I
\end{bmatrix},
\]

where \( \Delta = (1-L) \) is the first difference operator, \( \varepsilon_t = [\varepsilon_t^p, \varepsilon_t^I]' \) is a (2x1) vector of structural disturbances, and \( \Gamma(L) = \Gamma_0 + \Gamma_1 L + \Gamma_2 L^2 + \cdots \). The long-run multiplier matrix of the Solow-Swan model can easily be obtained from (5):

\[
\Gamma(1) = \sum_{i=0}^\infty \Gamma_i = \begin{bmatrix}
\delta Q_p^{-1}(1) & 0 \\
\delta Q_p^{-1}(1) & 0
\end{bmatrix}.
\]

Similarly, the long-run multiplier matrix for the AK model is derived from (7) as:

\footnote{See also Appendix A.}
In the Solow-Swan exogenous model, the first column of the long-run multiplier matrix is non-zero and the second column is zero. A productivity shock produces permanent effects on the levels of the variables, but a shock to investment share only causes transitory effects. In the AK endogenous model, both columns of the long-run multiplier matrix are non-zero. The effects of shocks to productivity and investment share are all permanent. The prediction that the transitory investment share shock exerts permanent effects on per-capita output is a hallmark of the AK-style models of endogenous growth. This stands in marked contrast to the Solow-Swan growth model, in which the long-run level of per-capita output depends solely on the process of productivity shocks. Therefore, the long-run effect of a transitory shock in investment share on per-capita output provides a test for evaluating the empirical adequacy of Solow-Swan exogenous and AK endogenous growth models.  

Equations (9) and (10) suggest that there is also commonality between the two models of

\[
\Gamma(1) = \sum_{i=0}^{\infty} \Gamma_i = \left[ \frac{sA}{1-\delta+sA} Q_{P1}(1) \right]^{Q_{P1}(1)} \left[ \frac{sA}{1-\delta+sA} Q_{I1}(1) \right]^{Q_{I1}(1)}.
\]

\[\text{Equations (9) and (10) suggest that there is also commonality between the two models of}\]

\[\text{In the class of endogenous growth models, other types of shocks besides investment share may potentially have permanent effects on per-capita output. For example, Kocherlakota and Yi (1996) examined whether temporary changes in government policies, i.e., income tax rates, import tariff rates, government expenditures, and growth rate of money supply, affect the long-run level of per-capita U.S. GNP. The survey paper by Grossman and Helpman (1991) provides a summary list of potential determinants of endogenous growth. Stadler (1986) and King et al. (1988) showed that within the context of an endogenous model, any transitory shock can cause permanent effects on output, as long as it produces temporary changes in the amount of resources allocated to growth. Nonetheless, the use of AK models may well serve as a representative for determining the empirical relevancy of endogenous models because the investment variable is present in most, if not all, growth models (Lau, 2008). Jones (1995) suggests that a test based on investment data may be able to address the endogenous and exogenous growth debate for an entire class of models in the literature.}\]
growth. For each structural shock, the long-run effects on the variables are identical. The reason is that log per-capita output and log per-capita investment are cointegrated with a coefficient vector of $[1, -1]$. This is a stochastic interpretation of balanced-growth constraints. As log per-capita output and log per-capita investment share a common trend, the great ratio – the difference between the two – becomes a stationary stochastic process along the steady-state growth path (for details, see King et al., 1991; Neusser, 1991; Fama, 1992). One econometric implication is that a vector error correction (VEC) model needs to be specified, which constitutes a starting point for the estimation of parameters in the following section.

3. Empirical models

This section presents the procedure that will be in use for the empirical evaluation of Solow-Swan exogenous and AK endogenous growth models in (9) and (10). Let $X_t = [\ln y_t, \ln i_t]'$, where $\ln y_t$ and $\ln i_t$ are $I(1)$ processes. Since the Solow-Swan and AK models posit one cointegrating relationship with a coefficient vector of $[1, -1]$, the error correction term $z_t$ is given as:

$$z_t = \beta'X_t = \ln y_t - \ln i_t,$$

(11)

where $\beta = (\beta_y', \beta_i')' = [1, -1]'$ is the cointegrating vector of dimension (2x1). The VEC model may be expressed as:
\( \Phi(L) \Delta X_t = \text{constant} + \alpha z_{t-1} + e_t, \quad (12) \)

where \( \Phi(L) = I - \Phi_1 L - \Phi_2 L^2 \cdots - \Phi_r L^r \), \( \alpha = [\alpha_y, \alpha_t]' \) is a (2x1) vector of error correction coefficients, and \( e_t = [e_{yt}, e_{zt}]' \) is a (2x1) vector of reduced-form disturbances with a mean of zero and covariance matrix \( \Omega \).

By employing the Wold decomposition and Granger’s Representation Theorem (Engle and Granger, 1987), the VEC model in (12) can be inverted into a reduced-form VMA formation as:

\[ \Delta X_t = \text{constant} + C(L)e_t, \quad (13) \]

where \( C(L) = I + C_1 L + C_2 L^2 + \cdots \). The long-run impacts of the reduced-form shocks on the levels of the variable are given by \( C(1) \). Specifically, as shown in Johansen (1991):

\[ C(1) = \sum_{i=0}^{\infty} C_i L^i = \beta_{\perp}' \gamma \alpha_{\perp}', \quad (14) \]

where \( \alpha_{\perp} \) and \( \beta_{\perp} \) are (2x1) vectors of the orthogonal complements to \( \alpha \) and \( \beta \), respectively, i.e., \( \alpha_{\perp}' \alpha = 0 \) and \( \beta_{\perp}' \beta = 0 \). \( \gamma \) is given by \((\alpha_{\perp}' \Phi(1) \beta_{\perp})^{-1}\), where \( \Phi(1) = I - \sum_{i=1}^{r} \Phi_i \) is a (2x2) full rank matrix from (12). The comparison between (8) and (13) reveals the relationships between the reduced-form and structural parameters:

\[ e_t = \Gamma_0 e_t \quad (15a) \]

and

10
Combining (13), (15a), and (15b) yields the corresponding structural VMA representation:

\[ \Delta X_t = \text{constant} + C(L)e_t = \text{constant} + C(L)\Gamma_0^{-1}e_t = \Gamma(L)e_t. \]  \hspace{1cm} (16)

The dynamic factor model we propose for empirical analysis has its origin in Stock and Watson (1988), Johansen (1991), Kasa (1992), and Escribano and Peña (1994). Specifically, it is assumed that

\[ \beta' \sim \mathcal{N}(0, \Omega), \] \hspace{1cm} (17)

and its inverse is given as:

\[ \Omega = [\beta' \beta']^{-1} \beta(\beta' \beta)^{-1}. \] \hspace{1cm} (18)

From (16) and (17), \( \beta' e_t \) and \( \beta e_t \) are permanent and transitory shocks, respectively, and their impacts on the variables over time are summarized in \( \Gamma(L) = C(L)\Gamma_0 \). A key feature is that the long-run effects of a transitory shock are not restricted to zero, but they are determined by data.

This can be verified using (14), (16), and (18), such that:

\[ \Gamma(1) = C(1)\Gamma_0 = [\gamma' \beta' \beta]^{-1} \beta(\beta' \beta)^{-1}, \] \hspace{1cm} (19)

where elements in \( \beta' \gamma' \beta(\beta' \beta)^{-1} \) can be any value.

We modify (17) in such a way that the structural shocks have an identity covariance matrix, i.e., \( E(e_t'e_t) = I \). This can be accomplished by assuming that:

\[ C(L)\Gamma_0 = \Gamma(L). \] \hspace{1cm} (15b)
\[
\Gamma_0^{-1} = \begin{bmatrix}
\Lambda_1 \beta_{-} \\
\Lambda_2^{-1} \beta \Omega^{-1}
\end{bmatrix},
\]
(20)

where \(\Lambda_1^{-1} \Lambda_1^{\top} = \beta_{-} \Omega \beta_{-}\) and \(\Lambda_2 \Lambda_2^{-1} = \beta \Omega^{-1} \beta\). It is straightforward to check that:

\[
E(e_t e_t') = \Gamma_0^{-1} E[e_t e_t'] \Gamma_0^{-\top} = \Gamma_0^{-1} \Omega \Gamma_0^{-\top} = 1.
\]

\(\Lambda_1 \beta_{-} e_t\) and \(\Lambda_2^{-1} \beta \Omega^{-1} e_t\) become the permanent and transitory shocks, respectively. The inverse matrix of \(\Gamma_0^{-1}\) is:

\[
\Gamma_0 = [\Omega \beta_1^{-1} \Lambda_1 \beta_2^{-1}, \beta_2^{-1}].
\]
(21)

The long-run multiplier matrix of structural shocks is calculated from (14) and (21) as:

\[
\Gamma(1) = C(1) \Gamma_0 = [\beta_1 \gamma \alpha_{-} \Omega \beta_{-} \Lambda_1 \beta_2 \gamma \alpha_{-} \beta \Lambda_2^{-\top}].
\]
(22)

As the implied cointegrating vector of \(\beta = [1, -1]'\) gives \(\beta_{-} = [1, 1]'\), \(\Gamma(1)\) of dimension (2x2) becomes:

\[
\Gamma(1) = \begin{bmatrix}
\gamma \alpha_{-} \Omega \beta_{-} \Lambda_1 \beta_2 \gamma \alpha_{-} \beta \Lambda_2^{-\top} \\
\gamma \alpha_{-} \Omega \beta_{-} \Lambda_1 \gamma \alpha_{-} \beta \Lambda_2^{-\top}
\end{bmatrix}.
\]
(23)

Equation (23) encompasses the Solow-Swan exogenous and AK endogenous growth models in (9) and (10). This feature presents a test for distinguishing the two growth models empirically:

The determining factor is whether or not \(\gamma \alpha_{-} \beta \Lambda_2^{-\top}\) is zero. If estimated to be zero, the evidence is taken to support the Solow-Swan model, in which a shock to investment share exhibits only transitory effects. If estimated to be non-zero, a shock to investment share produces permanent effects, and, therefore, the AK model is supported empirically.
Lau (2008) suggests an alternative way of testing the empirical veracity of exogenous and endogenous growth models. He assumes that $\Gamma_0$ in (15b) is a lower triangular matrix and then proves that if $\alpha_y$ in (12) is zero, the long-run effect of a transitory shock to investment share is zero in support of the exogenous growth model. This proposition can be easily understood by noting that when $\alpha_y = 0$, $\alpha^*_\perp = [1 \ 0]$ (see Fisher and Huh (2007) for detailed derivation). It follows that (14) is written as $C(1) = [\beta \gamma \ 0]$. Because $\Gamma_0$ is lower triangular, $C(1)\Gamma_0 = \Gamma(1)$ of (15b) yields that $\Gamma_{12}(1) = \Gamma_{22}(1) = 0$, implying no long-run effect of a shock to investment share. If $\alpha_y \neq 0$, $\Gamma_{12}(1) = \Gamma_{22}(1) = 0$ is refuted, and the endogenous model is chosen. While the Lau test is easy to implement, it is only valid under the assumption that a shock to investment share does not have a contemporaneous effect on per-capita output.

In this paper, we also consider the Beveridge and Nelson (1981) type decomposition of permanent and transitory shocks. The model posits that:

$$
\Psi_1 \alpha^*_\perp = \Psi_2 \alpha^* \Omega^{-1},
$$

where $\Psi_1^{-1} = \alpha^*_\perp \Omega \alpha_\perp$ and $\Psi_2 = \alpha^* \Omega^{-1} \alpha$ (Mellander et al., 1992; Englund et al., 1994; Fisher et al., 2000). The covariance matrix of the structural shocks is $E(\varepsilon_t \varepsilon_t^\top) = \Gamma_0^{-1} = \Gamma_0^{-1} \Omega \Gamma_0^{-1} = I$. The permanent and transitory shocks are given by $\Psi_1 \alpha^*_\perp \varepsilon_t$ and $\Psi_2 \alpha^* \Omega^{-1} \varepsilon_t$, respectively. The inverse matrix of $\Gamma_0^{-1}$ is:
\[ \Gamma_0 = [\Omega_\perp \Psi_1 \quad \alpha \Psi_2^{-T}] . \]  

(25)

From (16) and (25), the long-run multiplier matrix is given as:

\[ \Gamma(1) = C(1) \Gamma_0 = [\beta_\perp \gamma \Psi_1^{-1} \quad 0] = \begin{bmatrix} \gamma \Psi_1^{-1} & 0 \\ \gamma \Psi_1^{-1} & 0 \end{bmatrix} , \]  

(26)

where the last expression is obtained with the imposition of \( \beta_\perp = [1, 1]' \). Equation (26) is consistent with the exogenous growth model because a shock to investment share produces only transitory effects. Hence, it can be used to gauge how significantly the results from (23) differ from those implied by the exogenous growth model.

### 4. Empirical results and discussion

#### 4-1. Data description

Empirical analysis is undertaken with the quarterly data of output and investment in logarithms for the G-7 countries. The measure of output is GDP. Both the output and investment series were initially collected in nominal terms; they are divided by the GDP deflator and then divided by population to arrive at real per-capita values. Quarterly data on population were obtained by interpolating the annual series through the procedure INTERPOL in RATS. The sample period is 1960:1 to 2006:4, with the exception that France begins in 1970:1 due to data availability. All data were drawn from the IMF’s *International Financial Statistics* (IFS).
4-2. Test results

To check the stationarity of the data, the order of integration of the series is examined using the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS, 1992) test; the results are reported in Table 1. In all cases, the KPSS test rejects the null hypothesis of stationarity for the levels of the series at the 5% significance level. When the series are first differenced, the null hypothesis of stationarity cannot be rejected, implying that log per-capita output and log per-capita investment are characterized as $I(1)$ processes. This is consistent with the Solow-Swan exogenous and AK endogenous models of growth described in Section 2.

We then employ the dynamic OLS (henceforth DOLS) procedure of Stock and Watson (1993) to test whether log per-capita output and log per-capita investment are cointegrated with a coefficient vector of $[1, -1]$.\(^7\) The DOLS procedure provides estimates of the coefficients and their standard errors. It is also legitimate to make a statistical inference of the cointegrating coefficients in the usual manner. Yet, in order to establish that the equation estimated by DOLS is, in fact, a cointegrating relationship, its residuals must be shown to be stationary. The KPSS test

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\(^7\) The DOLS approach has some advantages over the Johansen (1991) maximum likelihood procedure. The Johansen method, being a full information technique, is exposed to the problem that parameter estimates in one equation are affected by any misspecification in other equations. DOLS, by contrast, is a single equation approach that corrects the potential of endogeneity and small-sample bias by the inclusion of leads and lags of the first differences of the regressors, and for serially correlated errors by using GLS. Stock and Watson show, through Monte Carlo studies, that DOLS is more favorable, particularly in small samples, compared to a number of alternative estimators including that of Johansen.
is applied to the residuals from the DOLS regression, and the critical values tabulated by Shin (1994) are used.\(^8\) Table 2 presents the results of the test. The number of lags/leads in the DOLS regression was determined using the Schwarz criterion. The estimated cointegrating vectors are normalized on log per-capita output (i.e., \(\beta_y = 1\)).

The table shows that the KPSS test does not reject the null hypothesis of a cointegration between log per-capita output and log per-capita investment at the 5% level of significance. The evidence is robust across the G-7 countries. In all cases, the coefficient for \(\ln i_t \ (\beta_i)\) is also estimated to be close to the theoretical value of \(-1\). The t-test confirms this result, as it cannot reject the null hypothesis of \(\beta_i = -1\). Therefore, log per-capita output and log per-capita investment are cointegrated with a coefficient vector of \([1, -1]\), as implied by balanced-growth constraints. We applied the cointegration test of Horvath and Watson (1995) as a check for robustness. This test examines the null hypothesis of no cointegration against the alternative of cointegration with a prespecified cointegrating vector. According to their Monte Carlo simulations, it possesses better power properties in comparison to other competitors that do not assume a cointegrating vector. Table 2 shows that the results remain the same. For all G-7 countries, the null hypothesis is rejected at the 5% level in favor of the cointegration relationship between the two variables with the coefficient vector of \([1, -1]\).

\(^8\) Note that the standard critical values of the KPSS test become invalid since the appropriate critical values depend on the number of regressors in the cointegrating regression.
Once the presence of cointegration is affirmed, VEC models are constructed in which the error correction term is \( z_t = \ln y_t - \ln i_t \). The number of lags in estimation was chosen using the Schwarz criterion. The estimated VEC models are expanded to models in the levels of the series. They are then inverted numerically to generate estimates of the reduced-form shocks. After these have been obtained, the structural shocks are identified, and their impacts on the series are calculated by utilizing the procedures outlined in Section 3. At issue is the long-run response of log per-capita output to a shock in investment share estimated from the dynamic factor (henceforth DF) model. If it is zero, the evidence supports the exogenous growth model. If it is significantly different from zero, the endogenous growth model is favored. Figure 1 depicts the responses of up to 100 quarters together with 95% confidence bands generated using 500 bootstrap replications. For comparison, the corresponding figures estimated from the structural Beveridge and Nelson (henceforth BN) model are also reported. This model assumes that the shock to investment share produces only transitory effects on log per-capita output, consistent with the exogenous growth model.

The results for Canada, Japan, the U.K., and the U.S. provide empirical support for the endogenous growth model. A positive shock to investment share leads to permanent increases in log per-capita output. The long-run effects are large and statistically different from zero at the 5% level. In the cases of Canada and Japan, all responses are significantly different from zero across
the horizons. Their 95% confidence bands also exclude responses from the BN model, confirming that the responses are empirically different from those of the exogenous growth model associated with the BN model. The results remain virtually unchanged for the U.K. and the U.S. The only exception is that respective responses from the DF and the BN models are statistically indistinguishable at some initial horizons.

In contrast, results for France, Germany, and Italy support the exogenous growth model. The long-run effects on log per-capita output of a shock to investment share are small and not different from zero at the 5% level. In the case of Italy, the long-run responses appear to be larger than zero, but only marginally significant. Germany shows particularly strong support for the exogenous growth model. The responses from the DF are statistically indistinguishable from those of the BN model across all horizons. The evidence is slightly weaker for France and Italy, but the main finding is the same. At long horizons, the 95% confidence bands constructed from the DF model include the responses from the BN model.

Table 3 presents the fraction of the forecast error variance of log per-capita output attributable to a shock in investment share calculated from the DF model. Subtracting this from 100 yields the fraction that is attributable to a shock in productivity. The corresponding figures obtained by the BN model are reported alongside. The DF model indicates that a shock to investment share is a major determinant of long-run variation in log per-capita output for Canada,
the U.K. and the U.S.; it accounts for between 47% and 70% of the forecast error variance over the horizon of 100 quarters. This shock also explains a significant portion of long-run movements in Japan’s log per-capita output. These results are significantly different from those of the BN model, in which the long-run contribution of the investment share shock is restricted to zero by construction. This is to reinforce the result that Canada, Japan, the U.K., and the U.S. support the endogenous growth model.

For France and Germany, the DF model indicates that a shock to investment share explains a statistically insignificant fraction of the forecast error variance of log per-capita output at long horizons. This is also consistent with our preceding results that support the exogenous growth model. The evidence for Italy is, however, mixed. On the one hand, the contributions of a shock to investment share to the long-run variability of log per-capita output are statistically different from zero, which is in line with the endogenous growth model. On the other hand, their 95% confidence bands include the fractions of the forecast error variance of log per-capita that are due to the investment share shock in the BN model. As the magnitudes between the DF and BN models are statistically indistinguishable in the long run, the exogenous growth model is favored. While it is difficult to make a clearcut case, the latter, assisted by the results from the impulse response analysis, seems more revealing; hence, the exogenous growth model may be a better description of the Italian data.
It is interesting to compare the results from this section with those obtained from the Lau (2008) test in Section 3, which uses the statistical relevancy of $\alpha_y = 0$ to discriminate between exogenous and endogenous models of growth. Table 4 reports the estimates for $\alpha_y$ and $\alpha_i$. The t-test shows that the coefficient $\alpha_y$ is not statistically different from zero in the G-7 countries, except for the U.K. These six countries favor the exogenous growth model; the U.K. is the only country receiving empirical support from the endogenous growth model. It is apparent that the Lau test produces different results for the three countries: Canada, Japan, and the U.S. While there are other reasons for this disparity, one may be associated with the test’s assumption that $\Gamma_0$ is a lower triangular matrix (see Section 3). Figure 1 shows that for these countries, the contemporaneous response of log per-capita output to a shock in investment share is statistically different from zero. This indicates that the lower triangularity of $\Gamma_0$ is inconsistent with the data, raising the possibility of producing misleading implications. Indeed, the problem would not occur if the contemporaneous responses are insignificant from zero in compliance with the assumption; France, Germany, and Italy serve as examples where this assumption is satisfied. Lau’s test and the DF model reach the same conclusions as shown in Table 4 and Figure 1.

4-3. Policy implications

Given that our focus is on the long-run output response to a shock in the investment share of
output, an immediate implication of the empirical results is that even among the G-7 countries the appropriate long-run growth strategy should be different. That is, when it comes to investment policy as a stimulus for long-run growth, it would be misguided to assume that there is a “one-size-fits-all” strategy for the developed countries. Investment could be more effective and conducive to long-run growth in some countries over others: for example, a policy focused on encouraging investment in the US is more likely to have a positive long-run impact on output than a similar policy in France. It is also worthwhile to note that countries whose long-run growth are unlikely to be affected by an increase in investment share are the continental European countries (France, Germany and Italy) while countries in the other group are English speaking (the U.K., the U.S. and Canada) and/or tend to have high-end manufacturing bases (Japan is a clear example in this case). Thus, our findings may suggest that an appropriate policy design to promote long-run growth should take into account the social, legal and possibly labour market institutions of the respective countries. This is consistent with the presumption that long-run returns from investment would not simply depend upon the size of investment relative to output but the complementarities associated with different policies and institutional characteristics.\textsuperscript{9} In particular, in countries with a significant long-run output effect of investment, policymakers need to be particularly concerned when there is a prolonged decrease in investment.

\textsuperscript{9} There is a caveat, however. The policy implications concerning our results are confined to investment policy. Other policies that might have long-run impacts on growth such as education, R&D and so forth would require separate analysis and interpretation, which is beyond the scope of this paper.
share. The Lost Decade in Japan may be exemplary in this regard.

5. A Monte Carlo experiment

This section examines the performance of the DF model using a Monte Carlo experiment.

Consider a two-variable VEC model of $\Delta X_t = [\Delta x_{1t}, \Delta x_{2t}]'$:

$$
\Delta X_t = \alpha z_{t-1} + e_t, \quad (27)
$$

where $\alpha$ is a (2x1) vector of error correction coefficients, $z_t = \beta' x_t = x_{2t} - x_{1t}$ is the error correction term, and $e_t$ is a (2x1) vector of reduced-form shocks. Subject to identification, a structural VMA representation corresponding to (27) is given as:

$$
\Delta X_t = \Gamma(L)e_t, \quad (28)
$$

where $e_t = [e_{1t}, e_{2t}]'$ is a (2x1) vector of structural shocks and is normally distributed with a mean of zero and an identity covariance matrix. The long-run multiplier matrix $\Gamma(1)$ may be written as:

$$
\Gamma(1) = \begin{bmatrix}
\zeta & 0 \\
\zeta & \theta
\end{bmatrix}, \quad (29)
$$

where $\zeta$ and $\theta$ are the parameters to be determined. This follows because $\beta = [-1, 1]'$ and $\beta' \Gamma(1) = 0$.\textsuperscript{10}

\textsuperscript{10}From (15b), $\beta' C(1) \Gamma_0 = \beta' \Gamma(1)$. Since $\beta' C(1) = 0$ by the Granger's Representation Theorem (Engle and Granger, 1987), $\beta' \Gamma(1) = 0$.\textsuperscript{10}
For the present paper, the issue is the long-run effect of $\varepsilon_{2t}$ on $x_{1t}$, denoted as $\theta$ in (29). We simulate the model for nine cases where $\theta=0$, $\pm 0.25$, $\pm 0.5$, $\pm 1$, and $\pm 1.5$. Details on the data generating process are discussed further in Appendix B. The total number of simulations is 5,000 and the sample size is 200. In each simulation, two sets of impulse responses are generated: One consists of true impulse responses; the other set of impulse responses is obtained by applying the DF model. They are compared to determine how exactly the responses from the DF model match the true ones at long horizons. For a formal test, we employ the Quasi-Lagrange Multiplier (Q-LM) procedure of Mittnik and Zadrozny (1993) and Nason and Cogley (1994). The Q-LM test statistic is distributed $\chi^2(1)$ for the null hypothesis that there is no statistical difference between the true and DF responses. Table 5 reports the percentage of replications in which the Q-LM statistics reject the null hypothesis at the 5% level of significance. The responses are at horizons of 24, 25, and 26.

Let us first look at the testing results for the case of $\theta = 0$, in which the true model exhibits zero long-run effect of $\varepsilon_{2t}$ on $x_{1t}$. The percentages of rejecting the null hypothesis are

---

11 Initially, 500 observations were generated: The first 300 observations were discarded to remove any dependence on initial values.
12 The test statistic takes the quadratic form of $[\Gamma - \hat{\Gamma}]\hat{\Sigma}^{-1}[\Gamma - \hat{\Gamma}]$, where $\Gamma$ is the true response, and $\hat{\Gamma}$ is the DF response. $\hat{\Sigma}$ is the covariance matrix of the DF response, which is calculated as $\hat{\Sigma} = \frac{1}{N} \sum_{i=1}^{N} [\hat{\Gamma}(i) - \hat{\Gamma}][\hat{\Gamma}(i) - \hat{\Gamma}]'$, where $\hat{\Gamma}(i)$ is the bootstrapped response of the DF procedure at replication $i$, and $N$ is the total number of bootstrap replications. $N=500$ is used for the application at hand.
13 The long-run responses of $x_{2t}$ are not reported, since they are the same as the long-run responses of $x_{1t}$ by the cointegration restrictions.
14 All impulse response estimates completed their empirical convergence to the long-run values by 23 horizons.
all zero for the long-run responses of $x_{1t}$ to $\varepsilon_{1t}$ and $\varepsilon_{2t}$. The DF model produces the responses that are statistically indifferent from the true responses, while it did not restrict the long-run effect of $\varepsilon_{2t}$ on $x_{1t}$ to being zero. The evidence remains virtually the same when $\theta$ takes non-zero values. The true responses are within the 95% confidence bands of the responses from the DF model. There are a few rejections in the cases of $\theta=1$ and $\theta=1.5$, but the effects are minor and do not alter the main finding. Overall, this experiment augurs well for the DF model in capturing the long-run responses of the model.

6. Concluding remarks

This paper has examined the empirical relevance of the Solow-Swan exogenous model and AK endogenous growth model for the G-7 countries by exploiting a key difference between the two: whether a transitory shock to investment share exhibits permanent effects on per-capita output. To test for this difference empirically, we develop a dynamic factor model that generates dynamic responses of the variables to permanent and transitory shocks. The long-run effects of a transitory shock in investment share are not restricted to being zero; they are determined from the data. This feature presents a test for distinguishing between the exogenous and endogenous models of growth empirically. A Monte Carlo experiment indicates that the dynamic factor model recovers the true long-run responses of the variables.
Our results suggest that the endogenous growth model receives more support in Canada, Japan, the U.K., and the U.S. A transitory shock to investment share leads to permanent increases in log per-capita output, and the long-run effects are large and statistically different from zero. In fact, most responses are significantly different from those of the estimated exogenous growth model across the horizons. The investment share shock is also equally important as a shock to productivity in accounting for the long-run variability of per-capita output. For the remaining three countries (France, Germany, and Italy), they are more consistent with the exogenous growth model than with the endogenous growth model.

In the literature, exogenous growth models tend to have greater empirical support from time series studies. Different results appear to emerge, however, when a more general model for empirical investigation is utilized. This is what the current paper shows. The endogenous growth model responds more favorably than previous studies suggested. This result has important policy implications because, contrary to the neoclassical presumption, raising investment share even temporarily can have a significant long-run effect on per capita output in a number of countries. Growth-focused policymakers in these countries may need to be particularly concerned when the investment to output ratio is falling. Furthermore, the results from our time series approach are in line with cross-section studies that provide empirical support for endogenous growth models. While a more comprehensive analysis is warranted, it appears that there may be a possibility of
reconciling the weak support thus far obtained in the time series analysis for endogenous models with the results from cross-section studies.

Our study provides consideration for future research. First, it is possible that government policy choices can lead to temporary shocks to the investment share of output; however, our approach is not designed to address the sources of such variation.\textsuperscript{15} Second, like previous studies in the literature, we have focused on one particular testable implication of the exogenous versus endogenous growth models. Given these limitations in the current empirics of growth models, future studies may focus on evaluating multiple dimensions of growth models in a single econometric framework that can utilize both time series and cross-sectional data.

\textsuperscript{15} For example, both technological change and government policy choices can influence shocks to investment share, but differentiating between these sources is beyond the scope of the current empirical study, as it requires shocks to be endogenous: a challenging econometric issue.
Appendix A: Derivation of the equations (5) and (7)

For succinct exposition, the derivation makes a strict reference to the equations shown in the main section of the paper.

1. Deriving equation (5) from the neoclassical exogenous growth model

Divide through equations (1), (2) and (4) all by \((1+\tau)N\) to transform the equations in terms of the variables in efficiency units of labour, respectively, to get

\[
\ddot{y}_t = A\ddot{k}_t^2 \eta_t^\rho \\
\ddot{i}_t / \ddot{y}_t = s\eta_t' \\
(1 + \tau)\ddot{k}_{t+1} = (1 - \delta)\ddot{k}_t + \ddot{i}_t
\]

where

\[
\ddot{y}_t = \frac{Y_t}{(1+\tau)N}, \quad \ddot{k}_t = \frac{K_t}{(1+\tau)N} \quad \text{and} \quad \ddot{i}_t = \frac{I_t}{(1+\tau)N}.
\]

Taking logs of (1)* and (2)* gives

\[
\ln \ddot{y}_t = \ln A + \lambda \ln \ddot{k}_t + \ln \eta_t^\rho \\
\ln \ddot{i}_t - \ln \ddot{y}_t = \ln s + \ln \eta_t'
\]

(A-1) (A-2)

To log-linearise (4)*, first divide both sides of the equation by \(\ddot{k}_t\)

\[
(1 + \tau)\ddot{k}_{t+1} / \ddot{k}_t = (1 - \delta) + \ddot{i}_t / \ddot{k}_t
\]

And then divide through by \((1+\tau)\) and re-arrange it to get

\[
\ddot{k}_{t+1} / \ddot{k}_t - (1 - \delta) / (1 + \tau) = (1 + \tau)^{-1} (\ddot{i}_t / \ddot{k}_t)
\]

Using the technique used by Campbell (1994), take logs on both sides and using the fact that for
a variable $X_t$, $X_t = \exp(x_t)$, where $x_t = \ln X_t$, we get

$$\ln \left[ \exp \left( \ln \left( \frac{k_{t+1}}{k_t} \right) - \frac{1-\delta}{1+\tau} \right) \right] = \ln \left( 1 + \tau^{-1} \left( \frac{i_t}{k_t} \right) \right)$$

Given that the LHS is a function of $\tilde{k}_{t+1} / \tilde{k}_t$, use a first-order Taylor approximation around the steady state value (N.B. $\tilde{k}_t = \tilde{k}_{t+1} = \tilde{k}$ in the steady state) to obtain

$$\text{LHS} \approx \left( \frac{1+\tau}{\delta+\tau} \right) \ln \left( \frac{\tilde{k}_{t+1}}{\tilde{k}_t} \right)$$

Hence, the linearised equation around the steady state is

$$\left( \frac{1+\tau}{\delta+\tau} \right) \ln \left( \frac{\tilde{k}_{t+1}}{\tilde{k}_t} \right) = -\ln(1+\tau) + \ln \tilde{i}_t - \ln \tilde{k}_t$$

Dating backward by one period and then using the lag operator, such that, for a variable $X_t, L^X_{t+1}$

$$= X_t,$$ we get

$$\left[ \left( \frac{1+\tau}{\delta+\tau} \right) - \left( \frac{1-\delta}{\delta+\tau} \right) L \right] \ln \tilde{k}_t = -\ln(1+\tau) + \ln \tilde{i}_{t-1}$$

(A-4)

Now take (A-1), $\ln \tilde{y}_t = \ln A + \lambda \ln \tilde{k}_t + \ln \eta^\rho_t$. Divide it through by $\lambda$ and then multiply both sides by $\left[ \left( \frac{1+\tau}{\delta+\tau} \right) - \left( \frac{1-\delta}{\delta+\tau} \right) L \right]$ to substitute out $\ln \tilde{k}_t$, to write as

$$\frac{1}{\lambda} \left[ \left( \frac{1+\tau}{\delta+\tau} \right) - \left( \frac{1-\delta}{\delta+\tau} \right) L \right] \ln \tilde{y}_t = \frac{1}{\lambda} \left[ \left( \frac{1+\tau}{\delta+\tau} \right) - \left( \frac{1-\delta}{\delta+\tau} \right) L \right] \ln A - \ln(1+\tau) + \ln \tilde{i}_{t-1}$$

$$+ \frac{1}{\lambda} \left[ \left( \frac{1+\tau}{\delta+\tau} \right) - \left( \frac{1-\delta}{\delta+\tau} \right) L \right] \ln \eta^\rho_t$$

Dividing this through by $\frac{1}{\lambda} \left( \frac{1+\tau}{\delta+\tau} \right)$ we get
\[
\left[1 - \frac{1 - \delta}{1 + \tau} L\right] \ln \tilde{y}_t = \left(\frac{\delta + \tau}{1 + \tau}\right) \ln A - \lambda \left(\frac{\delta + \tau}{1 + \tau}\right) \ln(1 + \tau) + \lambda \left(\frac{\delta + \tau}{1 + \tau}\right) \ln \tilde{i}_{t-1} + \left[1 - \frac{1 - \delta}{1 + \tau} L\right] \ln \eta^p_t
\]

Taking (A-2), \( \ln \tilde{i}_t - \ln \tilde{y}_t = \ln s + \ln \eta^*_t \), along with the above equation, we can write them as a system of two equations in the vector \( [\ln \tilde{y}_t \ \ln \tilde{i}_t]^T \), to arrive at the following matrix system

\[
\begin{bmatrix}
1 - \frac{1 - \delta}{1 + \tau} L & -\lambda \left(\frac{\tau + \delta}{1 + \tau}\right) L \\
-1 & 1
\end{bmatrix}
\begin{bmatrix}
\ln y_t - \tau t \\
\ln i_t - \tau t
\end{bmatrix}
= \begin{bmatrix}
\left(\frac{\tau + \delta}{1 + \tau}\right) \ln \left(\frac{A}{(\tau + \delta)^\lambda}\right) \\
\ln s
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
1 - \frac{1 - \delta}{1 + \tau} L \ln \eta^p_t \\
\ln \eta^*_t
\end{bmatrix}
\]

Note that \( \ln \tilde{y}_t = \ln \left(\frac{y_t}{(1 + \tau)^\lambda}\right) = \ln y_t - \tau t \), \( \therefore \ln(1 + \tau) \approx \tau \).

To write the above (2×2) system of structural form (SF) equations in terms of a reduced form (RF) system, solve for \( \ln y_t - \tau t \) and \( \ln i_t - \tau t \), respectively, to get

\[
\begin{bmatrix}
1 - \left(\frac{(1 + \tau) - (1 - \lambda)(\tau + \delta)}{1 + \tau}\right) L \\
1 - \left(\frac{(1 + \tau) - (1 - \lambda)(\tau + \delta)}{1 + \tau}\right) L
\end{bmatrix}
\begin{bmatrix}
\ln y_t - \tau t \\
\ln i_t - \tau t
\end{bmatrix}
= \begin{bmatrix}
\left(\frac{\tau + \delta}{1 + \tau}\right) \ln \left(\frac{s^\lambda A}{(\tau + \delta)^\lambda}\right) \\
\left(\frac{\tau + \delta}{1 + \tau}\right) \ln \left(\frac{s^\lambda A}{(\tau + \delta)^\lambda}\right)
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
1 - \frac{1 - \delta}{1 + \tau} L \ln \eta^p_t \\
\frac{1 - \delta}{1 + \tau} L \ln \eta^p_t + \lambda \left(\frac{\tau + \delta}{1 + \tau}\right) L \ln \eta^*_t
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
1 - \left(\frac{(1 + \tau) - (1 - \lambda)(\tau + \delta)}{1 + \tau}\right) L \\
1 - \left(\frac{(1 + \tau) - (1 - \lambda)(\tau + \delta)}{1 + \tau}\right) L
\end{bmatrix}
\begin{bmatrix}
\ln y_t - \tau t \\
\ln i_t - \tau t
\end{bmatrix}
= \begin{bmatrix}
\left(\frac{\tau + \delta}{1 + \tau}\right) \ln \left(\frac{s^\lambda A}{(\tau + \delta)^\lambda}\right) \\
\left(\frac{\tau + \delta}{1 + \tau}\right) \ln \left(\frac{s^\lambda A}{(\tau + \delta)^\lambda}\right)
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
1 - \frac{1 - \delta}{1 + \tau} L \ln \eta^p_t \\
\frac{1 - \delta}{1 + \tau} L \ln \eta^p_t + \left(1 - \frac{1 - \delta}{1 + \tau} L\right) \ln \eta^*_t
\end{bmatrix}
\]

Now the assumption that the productivity impulse is an \( I(1) \) process while the saving rate
impulse is an $I(0)$ process means $\pi^p = 1$ and $\pi^l = 0$, and hence the processes (3) become

\begin{equation}
(1 - L)Q_p(L) \ln \eta^p_t = \varepsilon^p_t \tag{A-3-1}
\end{equation}
\begin{equation}
Q_I(L) \ln \eta^l_t = \varepsilon^l_t \tag{A-3-2}
\end{equation}

Inverting (A-5-1) and (A-5-2) and then using the moving average form of (A-3-1) and (A-3-2) to substitute out $\ln \eta^p_t$ and $\ln \eta^l_t$, we can represent the reduced form system comprising (A-5-1) and (A-5-2) as

\begin{equation}
\begin{bmatrix}
(1 - L) \ln y_t \\
(1 - L) \ln i_t
\end{bmatrix} = \begin{bmatrix}
\tau \\
\tau
\end{bmatrix} + \begin{bmatrix}
1 - \left( \frac{(1 + \tau) - (1 - \lambda)(\tau + \delta)}{1 + \tau} \right) L \\
1 - \left( \frac{1 - \delta}{1 + \tau} \right) L
\end{bmatrix}^{-1} \begin{bmatrix}
\left( 1 - \frac{1 - \delta}{1 + \tau} L \right) Q_{p}^{-1}(L) \\
\left( 1 - \frac{1 - \delta}{1 + \tau} L \right) Q_{l}^{-1}(L) \\
(1 - L)\lambda \left( \frac{\tau + \delta}{1 + \tau} L \right) Q_{p}^{-1}(L) \\
(1 - L) \left( 1 - \frac{1 - \delta}{1 + \tau} L \right) Q_{l}^{-1}(L)
\end{bmatrix} \begin{bmatrix}
\varepsilon^p_t \\
\varepsilon^l_t
\end{bmatrix}
\end{equation}

Setting $\tau = 0$, the above system can be succinctly written as the following vector moving average (VMA) formation [equation (5) shown in the paper].

\begin{equation}
\begin{bmatrix}
(1 - L) \ln y_t \\
(1 - L) \ln i_t
\end{bmatrix} = (\delta(1 - \lambda)L)^{-1} \begin{bmatrix}
\delta L Q_{p}^{-1}(L) \\
\delta L Q_{l}^{-1}(L) \\
\lambda \delta (1 - L) L Q_{p}^{-1}(L) \\
\delta (1 - L) L Q_{l}^{-1}(L)
\end{bmatrix} \begin{bmatrix}
\varepsilon^p_t \\
\varepsilon^l_t
\end{bmatrix}
\end{equation}

2. Deriving equation (7) from the AK endogenous growth model

The AK model assumes the following production technology:

\begin{align*}
Y_t &= AK_t \eta^p_t
\end{align*}
Writing it in per capita terms and taking logs, we get

\[
\ln y_t = \ln A + \ln k_t + \ln \eta_t^p
\]

(A-6)

All other equations (2) through (4) remain the same as in the neoclassical model. The only difference is to choose the different balanced growth path when log-linearising (4). In the AK model, the balanced growth path with a constant population is

\[
\gamma_k = \gamma_y = sA - \delta
\]

Log-linearising the capital accumulation equation (4) around the balanced growth path, and then repeating the same process leads to the analogous system

\[
\begin{bmatrix}
1 - \left(1 - \frac{1 - \delta}{1 + sA - \delta}\right)L - \left(\frac{sA}{1 + sA - \delta}\right)\ln s
\end{bmatrix}
\begin{bmatrix}
\ln y_t \\
\ln i_t
\end{bmatrix}
= \begin{bmatrix}
\ln(1 + sA - \delta) - \left(\frac{sA}{1 + sA - \delta}\right)\ln s \\
\ln s
\end{bmatrix}
+ \begin{bmatrix}
1 - \left(1 - \frac{1 - \delta}{1 + sA - \delta}\right)L
\end{bmatrix}
\begin{bmatrix}
\ln \eta_t^p \\
\ln \eta_t^i
\end{bmatrix}
\]

Solving for \(\ln y_t\) and \(\ln i_t\), and then using the moving average versions of (A-3-1) and (A-3-2) above, we obtain the following system of equations (equation (7) shown in the paper),

\[
\begin{bmatrix}
(1 - L)\ln y_t \\
(1 - L)\ln i_t
\end{bmatrix}
= \begin{bmatrix}
\ln(1 - \delta + sA) \\
\ln(1 - \delta + sA)
\end{bmatrix}
+ \begin{bmatrix}
1 - \frac{1 - \delta}{1 - \delta + sA}LQ_{P}^{-1}(L)
\left(\frac{sA}{1 - \delta + sA}\right)Q_{I}^{-1}(L)
\end{bmatrix}
\begin{bmatrix}
\epsilon_t^P \\
\epsilon_t^I
\end{bmatrix}
\]
Appendix B: A further note on Monte Carlo experiment

For the model of (27) and (28), the following triangular VAR representation is used as a data generating process

\[ \Delta x_{1t} = \omega_1 z_t + \omega_2 z_{t-1} + \epsilon_{1t} \]  \hspace{1cm} (B-1)

\[ z_t = \omega_3 \Delta x_{1t} + \omega_4 z_{t-1} + \epsilon_{2t}. \]  \hspace{1cm} (B-2)

The structural VMA representation for (B-1) and (B-2) is given as:

\[ \begin{bmatrix} \Delta x_{1t} \\ z_t \end{bmatrix} = \Xi(L) \epsilon_t. \]  \hspace{1cm} (B-3)

Premultiplying (B-3) by a (2x2) matrix \( \Theta \) and equating it with (28) yield

\[ \Gamma(L) = \Theta \Xi(L), \]  \hspace{1cm} (B-4)

where

\[ \Theta = \begin{bmatrix} 1 & 0 \\ 1 & 1-L \end{bmatrix}. \]

Setting \( L=1 \) on (B-4) suggests the relationship between the two VMA representations in the long run that:

\[ \Gamma(1) = \begin{bmatrix} \zeta & \theta \\ \zeta & \theta \end{bmatrix} = \begin{bmatrix} \Xi(1)_{11} & \Xi(1)_{12} \\ \Xi(1)_{11} & \Xi(1)_{12} \end{bmatrix}, \]  \hspace{1cm} (B-5)

where \( \Xi(1)_{ij} \) is the (i, j) element of the long-run multiplier matrix \( \Xi(1) \). Equation (B-5) shows

\[ \text{Equation (B-5) shows} \]

\[ \text{See Campbell and Shiller (1988), Mellander et al. (1992), and Lau (2008) for transforming VEC models into triangular VAR counterparts.} \]
that $\zeta$ and $\theta$ of $\Gamma(1)$ can be obtained by estimating (B-1) and (B-2), and calculating (B-3).

In (B-1) and (B-2), $\omega_1$, $\omega_3$, and $\omega_4$ are all set to 0.3. The equations are then simulated to generate artificial data for different values of $\theta$. Nine cases are considered in the text: $\theta = -1.5$, $-1.0$, $-0.5$, $-0.25$, 0.0, 0.25, 0.5, 1.0, and 1.5. It is easy to derive the long-run effect of $\varepsilon_{2t}$ on $x_{1t}$ as:

$$\Xi(1)_{12} = \frac{\omega_1 + \omega_2}{(1 - \omega_4) - \omega_3(\omega_1 + \omega_2)} = \theta.$$  \hspace{1cm} (B-6)

Given $\omega_1$, $\omega_3$, and $\omega_4$, those nine cases of $\theta$ are provided by assuming that $\omega_2 = -2.209$, $-1.3$, $-0.712$, $-0.489$, $-0.3$, $-0.137$, 0.004, 0.238, and 0.424, respectively.
References


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Table 1: Tests for unit roots

<table>
<thead>
<tr>
<th>Country</th>
<th>$\ln y_t$</th>
<th>$\ln i_t$</th>
<th>$\Delta \ln y_t$</th>
<th>$\Delta \ln i_t$</th>
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</thead>
<tbody>
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<td>no trend</td>
<td>trend</td>
<td>no trend</td>
<td>trend</td>
</tr>
<tr>
<td>Canada</td>
<td>3.69*</td>
<td>0.65*</td>
<td>2.98*</td>
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<td>2.19*</td>
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</tr>
<tr>
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<td>0.46*</td>
<td>3.40*</td>
<td>0.17*</td>
</tr>
<tr>
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</tr>
<tr>
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<td>2.87*</td>
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</tr>
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</tr>
<tr>
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<td>0.13</td>
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<tr>
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<td></td>
<td></td>
<td>0.43</td>
<td>0.10</td>
</tr>
<tr>
<td>U.K.</td>
<td>3.80*</td>
<td>0.23*</td>
<td>3.29*</td>
<td>0.20*</td>
</tr>
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<td>0.06</td>
</tr>
<tr>
<td>U.S.</td>
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<td>3.01*</td>
<td>0.23*</td>
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<td>0.03</td>
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<tr>
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<td>0.03</td>
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</tbody>
</table>

A truncation lag is set at four. Critical values for the test are drawn from Kwiatkowski et al. (1992): They are 0.463 with no trend and 0.146 with a trend at the 5% significance level. An asterisk (*) indicates statistical significance at the 5% level.
<table>
<thead>
<tr>
<th>Country</th>
<th>Lags/Leads</th>
<th>$\beta_i$</th>
<th>t-test</th>
<th>KPSS test</th>
<th>HW test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>2</td>
<td>-1.110 (0.19)</td>
<td>-0.578</td>
<td>0.160</td>
<td>13.86*</td>
</tr>
<tr>
<td>France</td>
<td>2</td>
<td>-1.248 (0.41)</td>
<td>-0.604</td>
<td>0.108</td>
<td>11.28*</td>
</tr>
<tr>
<td>Germany</td>
<td>4</td>
<td>-1.174 (0.13)</td>
<td>-1.338</td>
<td>0.169</td>
<td>12.64*</td>
</tr>
<tr>
<td>Italy</td>
<td>2</td>
<td>-1.060 (0.34)</td>
<td>-0.176</td>
<td>0.165</td>
<td>15.91*</td>
</tr>
<tr>
<td>Japan</td>
<td>2</td>
<td>-1.007 (0.25)</td>
<td>-0.028</td>
<td>0.186</td>
<td>17.23*</td>
</tr>
<tr>
<td>U.K.</td>
<td>2</td>
<td>-1.088 (0.15)</td>
<td>-0.586</td>
<td>0.157</td>
<td>14.57*</td>
</tr>
<tr>
<td>U.S.</td>
<td>3</td>
<td>-1.022 (0.09)</td>
<td>-0.244</td>
<td>0.144</td>
<td>15.92*</td>
</tr>
</tbody>
</table>

The second column reports the number of lags/leads in the DOLS regression. The error term is assumed to have an AR(4) process. The third column reports the estimated coefficient on $\ln i_t$ by DOLS, where the cointegrating vector is normalized by $\ln y_t$ (i.e. $\beta_y = 1$). Figures in parentheses are the standard errors. The fourth column reports the t-test statistic for the null hypothesis that $\beta_i$ is equal to $-1$. The fifth column reports the KPSS test statistic for the null hypothesis of cointegration. The critical value is 0.221 at the 5% significance level (Shin, 1994, p. 100). The final column reports the Horvath and Watson test statistic for the null hypothesis of no cointegration against the alternative that log per-capita output and log per-capita investment are cointegrated with a coefficient vector of $[1, -1]$. The critical value is 10.18 at the 5% significance level (Horvath and Watson, 1995, p. 996). An asterisk (*) indicates statistical significance at the 5% level.
Table 3: Forecast error variance decompositions of log per-capita output

<table>
<thead>
<tr>
<th>Qtrs</th>
<th>Canada</th>
<th>France</th>
<th>Germany</th>
<th>Italy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DF</td>
<td>BN</td>
<td>DF</td>
<td>BN</td>
</tr>
<tr>
<td>1</td>
<td>61.8</td>
<td>2.1</td>
<td>51.3</td>
<td>9.1</td>
</tr>
<tr>
<td></td>
<td>(47~77)</td>
<td>(32~70)</td>
<td>(13~30)</td>
<td>(9~23)</td>
</tr>
<tr>
<td>2</td>
<td>48.5</td>
<td>1.2</td>
<td>51.4</td>
<td>9.2</td>
</tr>
<tr>
<td></td>
<td>(34~63)</td>
<td>(33~70)</td>
<td>(13~32)</td>
<td>(7~21)</td>
</tr>
<tr>
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<td>40.2</td>
<td>1.4</td>
<td>37.0</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>(26~54)</td>
<td>(22~52)</td>
<td>(8~34)</td>
<td>(2~20)</td>
</tr>
<tr>
<td>25</td>
<td>46.7</td>
<td>0.6</td>
<td>27.5</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>(32~61)</td>
<td>(12~43)</td>
<td>(2~43)</td>
<td>(1~30)</td>
</tr>
<tr>
<td>50</td>
<td>47.5</td>
<td>0.3</td>
<td>24.4</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>(31~64)</td>
<td>(6~43)</td>
<td>(0~54)</td>
<td>(5~50)</td>
</tr>
<tr>
<td>75</td>
<td>47.5</td>
<td>0.2</td>
<td>23.6</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>(29~66)</td>
<td>(2~45)</td>
<td>(0~60)</td>
<td>(11~64)</td>
</tr>
<tr>
<td>90</td>
<td>47.5</td>
<td>0.2</td>
<td>23.3</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>(28~67)</td>
<td>(0~46)</td>
<td>(0~60)</td>
<td>(13~70)</td>
</tr>
<tr>
<td>100</td>
<td>47.5</td>
<td>0.2</td>
<td>23.2</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>(28~67)</td>
<td>(0~47)</td>
<td>(0~61)</td>
<td>(14~74)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Qtrs</th>
<th>Japan</th>
<th>U.K.</th>
<th>U.S.</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>DF</td>
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<td>DF</td>
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</tr>
<tr>
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<td>0.3</td>
<td>31.1</td>
</tr>
<tr>
<td></td>
<td>(12~20)</td>
<td>(17~45)</td>
<td>(14~26)</td>
</tr>
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<td>27.3</td>
<td>1.4</td>
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<tr>
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</tr>
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<td>30.7</td>
<td>1.6</td>
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<td>(16~45)</td>
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<td>1.1</td>
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</tr>
<tr>
<td></td>
<td>(12~44)</td>
<td>(52~86)</td>
<td>(35~60)</td>
</tr>
</tbody>
</table>

Figures are the fractions of the forecast error variance of log per-capita output attributable to a shock in investment share estimated from the dynamic factor (DF) and Beveridge and Nelson (BN) models, respectively. Those in parentheses are 95% confidence bands generated using 500 bootstrap replications of the DF model.
Table 4: Tests for error-correction coefficients

<table>
<thead>
<tr>
<th>Country</th>
<th>lags</th>
<th>$\Delta \ln y_t$</th>
<th>$\Delta \ln i_t$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\alpha_y$</td>
<td>t-test</td>
</tr>
<tr>
<td>Canada</td>
<td>4</td>
<td>-0.226 (0.64)</td>
<td>-0.353 (0.64)</td>
</tr>
<tr>
<td>France</td>
<td>5</td>
<td>-0.517 (0.55)</td>
<td>-0.940 (0.97)</td>
</tr>
<tr>
<td>Germany</td>
<td>5</td>
<td>0.220 (0.97)</td>
<td>0.226 (0.22)</td>
</tr>
<tr>
<td>Italy</td>
<td>8</td>
<td>1.256 (1.35)</td>
<td>0.930 (0.93)</td>
</tr>
<tr>
<td>Japan</td>
<td>7</td>
<td>0.108 (1.05)</td>
<td>0.102 (0.10)</td>
</tr>
<tr>
<td>U.K.</td>
<td>4</td>
<td>1.704 (0.85)</td>
<td>2.004*</td>
</tr>
<tr>
<td>U.S.</td>
<td>6</td>
<td>0.954 (1.11)</td>
<td>0.859 (0.85)</td>
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</table>

The second column reports the number of lags in the VEC model. The third column reports the estimate for the error correction coefficient $\alpha_y$ in the output equation. Figures in parentheses are the standard errors. The fourth column reports the t-test statistic for the null hypothesis that $\alpha_y$ is equal to zero. The fifth and sixth columns do the same for the investment.
Table 5: Monte Carlo experiments

<table>
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<tr>
<th>Horizons</th>
<th>$\varepsilon_1 \rightarrow x_1$</th>
<th>$\varepsilon_2 \rightarrow x_1$</th>
<th>$\varepsilon_1 \rightarrow x_1$</th>
<th>$\varepsilon_2 \rightarrow x_1$</th>
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<td>$\theta = -1.5$</td>
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<td></td>
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</table>

Figures are the percentages of replications in which the Q-LM statistics reject the null hypothesis that there is no statistical difference between the true and DF responses at the 5% significance level.
Figure 1: Responses of log per-capita output to a one-standard-deviation shock in investment share (percent)
Figure 1 (Continued)

- Japan
- U.K.
- U.S.

- Dynamic factor model
- 95% confidence bands
- Beveridge and Nelson model